UNIT – I

NATURE OF MATHEMATICS

Structure

1.1 INTRODUCTION

Every student-teacher of Mathematics should become an effective teacher of Mathematics. So as student-teachers, you will have to be familiar with the definitions and the nature of Mathematics to become a good and efficient teacher of Mathematics.

Today we have tremendous advancement made in the area of Mathematics. It has wide application value to other sciences and other fields of human experiences. Hence one definition will not cover the entire Scope of Mathematics. This unit presents number of working definitions of Mathematics and some important Characteristics of Mathematics such as objectivity, symbolism, abstractness, logical structure, Applicability, Precision and Accuracy. By considering the development of mathematics and contributions of mathematicians

through the ages, you will be able to make your students appreciate the fact that mathematics has continually adjusted itself to human needs both materials and intellectual and to motivate them we should make an effort to improve the status of mathematics by adding to what exists now through their discoveries.

This unit brings out the history of mathematics and the contribution of various western and Indian mathematicians.

1.2 OBJECTIVES

At the end of the unit, the student-teachers will be able to

- Define mathematics.
- Acquire a clear perspective of the nature of mathematics.
- Identify the characteristics belonging to the nature of mathematics.
- Know the contributions of western and Indian mathematics.
- Understand the importance of history of mathematics for a teacher.

1.3 IMPORTANCE OF MATHEMATICS

In today's world, we are bombarded with data that must be absorbed, sorted, organized, and used to make decisions. The underpinnings of everyday life, such as making purchases, choosing insurance or health plans, and planning for retirement, all require mathematical competence. Business and industry need workers who can solve real-world problems, explain their thinking to others, identify and analyse trends in data, and use modern technology.As society becomes more technically dependent, there will be an increasing requirement for people with a high level of mathematical training.

Mathematics is a universal part of human culture. It is the tool and language of commerce, engineering and other sciences – physics, computing, biology etc. It helps us recognise patterns and to understand the world around us.

Furthermore, learning mathematics forces to learn how to think very logically and to solve problems using that skill. It also teaches to be precise in thoughts and words. Mathematics teaches life skills. It is difficult to find any area of life that isn't touched by mathematics.

Today's students must master advanced skills in mathematics, science, and technology to stay on track for college and for promising careers. Mathematics teaches ways of thinking that are essential to work and civic life.

1.4 MEANING AND DEFINITIONS OF MATHEMATICS

"Mathematics is the subject which provides an opportunity for the training of mind to close thinking, stirring up a sleeping and unstructured spirit". - Plato

Etymologically the term 'Mathematics' is derived from two Greek words: 'Manthanein' which means 'learning and 'Techne' which means 'an art or technique'. The dictionary meaning of mathematics is that 'it is either the science of number and space or the science of measurement, quantity and magnitude'. Mathematics is thus defined as the science of quantity, measurement and spatial relations. It is a systematised, organised and exact branch of science. It deals with quantitative facts, relationships as well as with problems involving space and form. It is a logical study of shape, arrangement, and quantity. Mathematics is defined in different ways by different authors. Let us examine a few of them.

According to new English Dictionary -"Mathematics - in a strict sense - is an abstract science, which investigates deductively, the conclusions implicit in the elementary conceptions of spatial, numerical relations."

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According to Locke - "Mathematics is a way to settle in mind a habit of reasoning".

According to Lindsay "Mathematics is the language of physical sciences and certainly no more marvelous language was ever created by the mind of man".

According to Courant and Robbins - "Mathematics is an expression of human mind that reflects the active will, the contemplative reason and the desire for aesthetic perfection. Its basic elements are logic and intuition, analysis and construction, generality and individuality".

According to Servais - "Mathematics is an abstract science - it is the science of abstraction. Learning mathematics is learning to abstract, to handle abstractions and to use it. "

According to Higgins - "Mathematics is a laboratory subject. The power and advantage of mathematics lie not in concrete manipulations,

but in abstract symbolic manipulations which is always the final goal that we seek".

Bertrand Russell (1901) defined mathematics as "The subject in which we never know what we are talking about, nor whether what we are saying is true", is fundamentally significant. Bell, discussing the merits of this definition in his "Queen of Sciences" says that,

(i) It emphasises the entirely abstract character of mathematics

(ii) It reduces all mathematics to postulation forms, and

(iii) It opposes the traditional definition of mathematics as the science of number, quantity and measurement".

Comte defined mathematics as "The science of indirect measurement"

According to Kant "Mathematics is the indispensable instrument of all physical researches"

Gauss stated "Mathematics is the queen of sciences and arithmetic is the queen of all mathematics"

Bacon said "Mathematics is the gateway and key to all sciences"

J.B. Shaw stated, "Mathematics is engaged, in fact, in the profound study of art and the expression of beauty".

According to Shaw there are four significant methods of mathematics which give more insight into the nature of mathematics.

- Scientific, leading to generalisations of widening scope.
- Intuitive, leading to an insight into subtler depths.
- Deductive, leading to a permanent statement and rigorous form.
- Inventive, leading to the ideal element, and creation of new realms.

Mathematics, therefore, is not only 'number work' or 'computation', but is more about forming generalisations, seeing relationships, and developing logical thinking and reasoning.

The National Policy on Education (NPE) (1986) stated "Mathematics should be visualized as the vehicle to train a child to think, reason, analyse and to articulate logically". Mathematics should be shown as a way of thinking, an art or form of beauty, and as human achievement.

Thus, from above definitions, we can conclude that

i) Mathematics is an abstract Science.

ii) Mathematics is a science that deals with number and space.

iii) Mathematics deals with quantitative facts and relationships.

iv) Mathematics deals with problem involving space and form.

v) Mathematics establishes various relationships between phenomena in space.

vi) Mathematics helps man to give exact interpretation to his ideas and conclusions.

vii) Mathematics explains that Science is by-product of our empirical knowledge.

viii) Mathematics involves man's high cognitive powers.

ix) Mathematics has its own tools like intuition, logical reasoning, analysis, construction, generalities and individuality.

x) Mathematics is a Science of Logical – reasoning

xi) Mathematics is a tool especially suited for dealing with scientific concepts.

Check Your Progress

Notes : a) Write your answers in the space given below

b) Compare your answer with the one given at the end of the unit

1. What is the meaning of mathematics?

   ……

   ……

2. Give any two definitions of mathematics

   ……

   ……

1.5 CHARACTERISTICS OF MATHEMATICS

1.5.1 Objectivity

Objectivity is a central philosophical concept which has been variously defined by sources. A proposition is generally considered to be objectively true when its truth conditions are met and are "mind-independent" that is, not met by the judgment of a conscious entity or subject.

Objectivity in mathematics is traditionally thought of as one of the more desirable necessities for its own credibility. The basic assumptions are, on the one hand, that any mathematical description of nature must be independent of the wishes, moods, and/or needs of an individual mathematician (or group of mathematicians), and on a more fundamental basis that nature itself is objective, because it is ruled by laws and not by the concerns or intentions of its inhabitants. The latter is highly critical in that the laws of nature are deemed to be inviolable, quantitative, inexorable, and general; i.e., they should not bend for a purpose.

By objective it means there is "universal agreement" (or at least near universal agreement). Everyone agrees that "1 + 1 = 2" In fact, mathematics is possibly the only discourse where there is a real sense of universal agreement. This is the reason mathematics is so successful in facilitating the study of so many diverse fields. One can argue that mathematics is the most successful language ever invented. Their existence is an objective fact, independent of our knowledge of them. Our mathematical knowledge is objective and unchanging because it's knowledge of objects external to us, independent of us, which are indeed changeless.

1.5.2 Logical Structure

Mathematics uses an unfailing logical structure. The steps in a problem have to be arranged in a logical sequence. Every step or statement to be mathematical must be supported by a proper reason. If we do not do this, then, even the ordinary equations may lead us to fallacies.

Example: 1. The cost of 100 bags is 20 rupees.

It is wrong to write 100 bags = 20rupees.

2. A box which is half open is half closed. If we write it in the form of an equation,

½ open = ½ closed

Or

Box Open = Box Closed

This is a mistake arising out of the use of a wrong logic.

In mathematics we cannot accept statements not supported by a proper reason. The answer alone cannot be the criterion. An answer may be correct but it must be supported by logical steps. This training to think in terms of finding a reason for every statement will stand in good stead while the individual has to face problems in life.

Check Your Progress

Notes : a) Write your answers in the space given below

b) Compare your answer with the one given at the end of the unit

3. Write a short on Objectivity in mathematics.

……

……

4. What do you mean by logical structure?

……

……

1.5.3 Abstractness

Abstractness is the basis of mathematics. It is true that our daily life problems have helped the growth of mathematical principles. Yet all the principles of mathematics are based on abstractness. Mathematical principles and truths can grow even without practical values. Great mathematicians in the past were very much attracted towards this nature of mathematics.

Even at the elementary stage, mathematics is at the abstract level. For example, number is an abstract concept. When we teach children about numbers we make use of fingers, dolls etc,. We say two and show two balls. Two stands for an abstract concept. Showing balls is a concrete activity. Concretization helps to develop abstractness. After a particular stage, students should leave the concrete level and try to learn mathematical facts and principles from the abstract level. When concrete experiences are not possible, the mathematical concepts, principles well understood by students serve as concrete foundation for further abstract level of mathematic learning.

All the numbers and operations such as 1,2,… and union [U] and intersection[∩] are abstractions; percent, length, volumes are abstractions; sum, difference, product, average are facts that concern elements or aspects which may appear with countless different concrete surroundings.

1.5.4 Symbolism

Mathematics has a unique language of symbols. It is a technical way of expressing a certain range of ideas in a form which is convenient for applying reasoning processes. The language of mathematics is applicable to a limited range of ideas with comparative degree. It has its own precision, conciseness an accuracy which are absent from any other language.

Every equation and inequality is a statement explaining a situation. =,<,> are the symbols they use. Teachers should use symbols clearly and correctly and train pupils to do the same. Symbols are the tools for conveying ideas. They are very useful for expressing a rule or principle briefly. We call this a formula.

$$A = \frac{n}{360} \times \pi r2, \quad A = \pi r2 \ldots\ldots\ldots\ldots$$

Some symbols in mathematics :0,1,2,3,…… =, +, -, ÷,>, <, √, ∩, ≥, ≤, ≡, ≠, ∑, π, θ, %,{},……

The special feature of mathematics is its symbolic language. In music we use different symbols for denoting sounds. So also in mathematics we use symbols to bring out clearly mathematical relationships.

Symbolic language has a definite purpose verbal explanations however clear they are; cannot convey an idea precisely. Symbolic language can achieve this. We know that the distributive law is. a(b+c) = ab + ac

To give a verbal explanation to this law, we have to use too many words which may be confusing. Again symbolic language helps one to remember formulate and rules.

A language of mathematics is much more than the word, symbols and the grammar it contains. The way all these components are put together provides a complete logical reasoning and thinking with which the concept principles and rules of mathematics are associated. Pupils learning mathematics should be able to distinguish the uses of symbols; because they represent basically three different kinds of meanings and understanding; namely

1. The elements or mathematical object: they are "numbers" represented by the symbols [numerals]; e.g.8, 17; sets such as A,B or geometrical objects such as lines 1, m which are used in mathematical sentences like 8<17; ACB=0,1/m etc.
2. The relationship between elements: The symbols like>,=,< etc. represent the verbs of mathematics. These express the particular way in which the elements are related to each other.
3. The operation on elements: The symbols like +,x, U,∩…represent operations performed on elements such as addition, multiplication, union and intersection respectively and brackets or parenthesis perform the function of punctuation in the language of mathematics as could be seen from the different answers for

$$64 \div [16 \div 4] = 16$$

$$[64 \div 16] \div 4 = 1$$

Thus, it is essential to insert the appropriate mathematical punctuation the bracket or parenthesis.

When we use these terms, signs of symbols, we must use only those in convention. Any mathematical statement should have the following characteristics.

a) It should make use of conventional signs and symbols.

b) Every word or symbol used in it should have a definite purpose.

c) Its language should be clear and unambiguous.

d) As far as possible, the statement should be simple and brief.

Check Your Progress

Notes : a) Write your answers in the space given below
b) Compare your answer with the one given at the end of the unit

5. How does concretisation help to develop abstractness?
……
……

6. List out the characteristics of a mathematical statement.
……
……

1.5.5Applicability

Knowledge is power only when it is applied. The study of mathematics requires the learners to apply the skill acquired to new situations. The knowledge acquired by the students is greatly used for solving problems. The students can always verify the validity of the mathematical rules and relationships by applying them to novel situations. The teachers should always help the students in applying and verifying the mathematical ideas. The knowledge and its application, wherever possible, should be related to daily life situations. Concepts and principles become more functional and meaningful only when they are related to actual practical applications. Such a practice will make the learning of mathematics more meaningful and significant.

1.5.6 Precision and Accuracy

Mathematics is known as an exact science because of its precision. It is perhaps the only subject which can claim certainty of results. In mathematics the results are either right or wrong, accepted or rejected. There is no midway possible between right and wrong. Mathematics can decide whether or not its conclusions are right. Mathematicians can verify the validity of the results and convince others of its validity with

consistency and objectivity. This holds not only for the expert, but also for anyone who uses mathematics at any level.

Even when there is a new emphasis on approximation, mathematical results can have any degree of accuracy required. Although precision and accuracy are distinctly different as criteria for the measures of approximation, they can be most effectively discussed when contrasted with each other. The most effective measures of both precision and accuracy are in terms of the errors (positive or negative) involved. The precision of a measure or a computation is evaluated in terms of the apparent error. The accuracy of a measure or a computation is evaluated in terms of the relative error or percent of error made.

It is the teacher's job to help the students in taking decisions regarding the degree of accuracy which are most appropriate for a measurement or calculation. This is possible by encouraging the students to observe critically, perceive relationships, analyse the data and arrive at precise conclusions/inferences to the level of accuracy required.

Check Your Progress

Notes : a) Write your answers in the space given below
b) Compare your answer with the one given at the end of the unit

7. What do you mean by precision in mathematics?

……

……

8. Write a short on Applicability.

……

1.6 HISTORY OF MATHEMATICS

The area of study known as the history of mathematics is primarily an investigation into the origin of discoveries in mathematicsand, to a lesser extent, an investigation into the mathematical methods and notation of the past.

Before the modern age and the worldwide spread of knowledge, written examples of new mathematical developments have come to light only in a few locales. The most ancient mathematical texts available are Plimpton 322 (Babylonian mathematics c. 1900 BC),the Rhind Mathematical Papyrus (Egyptian mathematics c. 2000-1800 BC) and the Moscow Mathematical Papyrus (Egyptian mathematics c. 1890 BC). All of these texts concern the so-called Pythagorean theorem, which seems to be the most ancient and widespread mathematical development after basic arithmetic and geometry.

The study of mathematics as a subject in its own right begins in the 6th century BC with the Pythagoreans, who coined the term

"mathematics" from the ancient Greek (mathema), meaning "subject of instruction". Greek mathematics greatly refined the methods (especially through the introduction of deductive reasoning and mathematical rigor in proofs) and expanded the subject matter of mathematics. Chinese mathematics made early contributions, including a place value system.The Hindu-Arabic numeral system and the rules for the use of its operations, in use throughout the world today, likely evolved over the course of the first millennium AD in India and were transmitted to the west via Islamic mathematics through the work of Muḥammadibn Musa al-Khwarizmi.Islamic mathematics, in turn, developed and expanded the mathematics known to these civilizations. Many Greek and Arabic texts on mathematics were then translated into Latin, which led to further development of mathematics in medieval Europe.

1.6.1 Western and Indian Mathematicians

The place which mathematics occupies today is made possible through dedicated and sustained hard work of its students through ages. Mathematics progressed all the way through the contribution of several Western and Indian Mathematicians. Through their invaluable contribution they have immortalised themselves. Introducing the biographies of outstanding mathematicians can inspire the students. Here, we are going to discuss the contributions of a few outstand mathematicians from abroad and India.

EUCLID

Euclid is believed to be an Egyptian who comes to Alexandria to learn and afterwards placed in charge of the mathematics department of the Alexandria University. His influence on mathematics began about 300 B.C. He wrote a textbook on Geometry, called the Elements, and it covers all the essential parts of mathematics known in his time. The Elements contains 13 books. Each text begins with definitions, postulates, and common opinions, then proceeds to obtain results by rigorous geometric proof. Books 1-6 deals with plane Geometry. Books 7-9 contain elements of numbers theory, where number means positive integers greater than 1, beginning with 22 new definitions - such as unity, even, odd and prime those books develop various properties of the positive integers. Books 10-13, examine three dimensional figures in Greek Stereometria. Euclid also wrote other books in mathematics and a few in physics. The important of them are ‘Date’, ‘On Division of Figures’, ‘Phenomena’, ‘Optics’ etc. In all his works Euclid follows the basic logical structure of Elements, having definitions and rigorously proved propositions. All works continue to exist in the Original Greek except ‘On Division of Figures’, which is partially preserved in Arabic. His major contributions are:

- Euclid’s second theorem - Prime numbers are infinite.

- He thought about the three current problems of time - Dividing an angle into three equal parts, Making double of a cube and obtaining square from a cube.
- Euclidean Algorithm-The greatest common divisor of two numbers.

PYTHAGORAS

Pythagoras was a Greek Mathematician. Pythagoras is believed to have been a pupil of Thales. He too had many followers. Pythagoras proved the position relating to the sum of the angels of a triangle and a few others. Though he is given credit for the theorem at least 1,500 years before, while the Chines knew it about 100 years previously, and the Hindus were also aware of it. We are still in doubt as to what method Pythagoras and his followers did much work on polyhedra which are special names according to the number of sides which make up the solid. Pythagoras also studied the properties of areas and volumes: and he was first to prove that a circle contains a greater areas than any plane figure with the same perimeter, while the sphere contains a greater volume than other shape bounded by the same surface. He, indeed, made geometry a science by basing it on axioms, postulates, and definitions and by setting down methods of proof. He was also one of the first men to class all numbers as even or add (Odd numbers were called gnomon). An odd number, which we call $2n + 1$ was looked upon as the difference of two square numbers $(n + 1)^2$ and n^2 or to illustrate this, $(3+1)^2-3^2$, where $n = 3$. He also states that the sum of the odd numbers from 1 to $(2n+ 1)$ was a square number, namely, $(n+1)^2$.

The square root of a square number was termed a side by the Pythagoreans, while the product of two numbers was called a plane, unless the product had no exact square root, when it was tanned an oblong. If the three same numbers were multiplied together, the product was called cube. He was very much impressed by numbers, and believed that they possessed different qualities, for example, number 1 stood for reason, 2 for opinion, 4 for justices, etc. Pythagoras also developed the theory of music. He was perhaps the most outstanding person in the history of ancient mathematics.

DESCARTES

Descartes was one of the greatest French mathematicians of the 17^{th} century. As he was weak in health, he did much of his mental work in bed. He was greatly interested in analytical geometry. He brought algebra and geometry together to form analytical geometry. He sought to make algebra clearer by applying geometrical method to it. He found that the position of any point in a plane could be found, if its distance called coordinates (x and y) are given or calculated form two fixed lines at right angles, called axes. He contributed a lot to the progress of exact

sciences. Descartes is also remembered for his 'Rule of Signs'. According to him an equation can have changes of sign from plus to minus, and no more false roots than the number of times two plus or two minus signs occur in succession. Take for example, an equation:

$$x^4 - x^3 - x^2 + x - 1 = 0$$

As this equation has three changes of sign, it cannot have more than three positive roots. Since two minus signs occur in succession only once, and two plus signs not at all, the equation can have only one negative root.

Aryabhatta

One of the best known of the Hindu writers was Aryabhatta. His chief work is divided into four parts, three, of which are on astronomy, while the fourth gives thirty three rules in Arithmetic, Algebra and Plane Geometry. In algebra, he deals with problems involving series. He derived the following formulae:

$$1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$1^3 + 2^3 + 3^3 + \ldots + n^3 = (1 + 2 + 3 + \ldots + n)^2$$

$$= \frac{n^2(n+1)^2}{4}$$

$$= \left[\frac{n(n+1)}{2}\right]^2$$

He was familiar with series, permutations and linear and quadratic equations in Algebra. He was also aware of the decimal system, for some of his arithmetic deals with numbers by tens upto 10^{18}. He also gave a rule for finding square roots and cube root along with their time in India the formula for interest, time and other related ones, in the problem. He gave accurate approximation for π, and wrote in his book. The following: Add four to one hundred, multiply by eight and then add sixty two thousand. The result is approximately the circumference of a circle of diameter twenty thousand. By this rule the relation of the circumference to diameter is given. This gives π = 62832/20000 = 3.1416 which is an accurate value of π. He gives formulae for the areas of a triangle and of a circle which arc correct. He introduces the versine (Versine = 1 - Cosine) into Trigonometry.

BHASKARACHARYA

Bhaskaracharya otherwise known as Bhaskara was one of the most powerful and creative mathematician of ancient India. He was also known as Bhaskara the learned. Well known works of Bhaskara are:

Lilavati, Bijaganita and Siddhantasiromani. Lilavati is about arithmetical and geometrical progressions, plane geometry, solid geometry, interest, permutations, ratio, etc., Bijaganita is about positive and negative numbers, zero, quadratic equations with more than one unknown and operations with products of several unknown, etc. Siddhantasiromani is about sphere, spherical trigonometry, ellipse calculations and astronomical calculations.

BRAHMAGUPTA

Brahmagupta wrote a book in verse dealing with arithmetic, algebra, and geometry. The arithmetic includes integers, fractions, progressions, simple interest, menstruation and some problems on simple plane geometry. His algebra is mainly applicable to astronomy. He gives the rules for negative numbers. He also gives the rules for solving quadratic equation of the type:

$x^2 + px + q = 0$. He also writes on simple simultaneous equations and solves some indetermined equations.

SRINIVASA RAMANUJAN

SrinivasaRamanujan was one of the greatest Indian mathematical geniuses. He was quite and meditative, and possessed an extraordinary memory. He delighted in entertaining his friends with theorems and formulae. He passed his matriculation examination at 16, but could not do well at the college. He was married in 1909, and it became necessary for him to find some permanent employment. He was appointed as a clerk in the office of the Madras Point Trust. But he did not Blacker his work in mathematics. His earliest contribution were published in the journal of Indian Mathematical Society. His article on 'Some Properties of Bernoulli's Numbers' way very much appreciated. Through the help of the chairman of the Madras Port Trust, he came in contact with the famous mathematician, C.H. Hardy. He got a scholarship from Madras and went to England.

Though he had no schooling in modern mathematics, but he could work out modular equations and theorems of complex multiplication to orders unheard of. His mastery of continued fractions was beyond that of any mathematicians in the world. He was a head of contemporary mathematical knowledge when he arrived in England. He recreated in his field through his own unaided powers a rich half century of European mathematics. One may doubt that so prodigious a feat had ever before been accomplished in the history of thought. Ramanujan was not a geometer; he cared nothing for mathematical physics, let alone the possible usefulness of his mathematical work to other disciplines. Numbers, as it appears, were his friends. In the simplest array of digits, he detected wonderful properties: Congruence, symmetries and relationships which has escaped the notice of even the outstandingly

gifted mathematicians. It was his insight into algebraical formulae, transformations of infinite series, etc., that was most amazing. 1729 is a very famous Ramanujan number. It is the smallest number which can be expressed as the sum of two cubes in two different ways i.e.

$1729 = 1^3 + 12^3 = 9^3 + 10^3$.

1.6.2 Importanceof History of Mathematics for a teacher

Mathematics holds the mirror uptocivilisation. It is no exaggeration to say that the history of mathematics is the history of civilisation. Mathematicians can take pride in the fact that their science, more than any other's is an exact science, and that hardly anything ever done in mathematics has proved to be useless. The geometry of the Greeks and the arithmetic of the Hindus are as useful and admirable as any research of today. Mathematics has been a progressive science.

Value of History

History of mathematics has not so far been given any place in its curriculum, simply because no time can be made available for its study when the already heavy courses have to be covered. Moreover, there has not been any serious realisation of the benefit which can possibly be had from this study. But once introduced, it is sure to become a source of interest and pleasure to the learners. Its values should be known to the mathematics teacher and the values can be explained as follows:

1. Mathematics will be presented a dynamic and progressive subject, full of human interest.
2. It will be instructive and interesting; it will not only remind us of what we have, but will also teach us how to increase our store of knowledge.
3. It warns the learner against hasty conclusions.
4. The teacher can better introduce many mathematical topics in the class by discussing their history.
5. It saves the student from attacking an unsolved problem by the same method which has led other mathematicians to failure.
6. It is important also as a valuable contribution to the history of civilisation. Mathematical and physical researches are a reliable record of intellectual progress. It is one of the large windows through which the philosophic eye look into past ages and traces the line of intellectual development.
7. It reveals that at every stage, major or significant development of mathematics was conditioned by human needs.

8. Most of the terms, concepts and convertions can be properly understood only by reference to their historical background.
9. If the teacher uses his knowledge of the history of mathematics in the classroom, the students form a good impression of his study and knowledge. This helps him in commanding respect.
10. Gradation of the subject, correlation of mathematics with other subjects and psychological and logical order of the subject-matter all these are done excellently.
11. It shows that mathematics is a man-made science. Its history will thus encourage the child also to contribute something to it.
12. It reveals that all the branches of mathematics were developed in relation to one another. So it guards the learner against compartmentalisation.
13. The teacher's narration of stories and events, mentioned occasionally, bring about a healthy change in the monotony of the classroom work.
14. It gives the impression, that mathematics has an intimate connection with other branches of knowledge, and should not be treated as an isolated subject.
15. It makes students appreciate the progress of man over the ages. They would like to read and hear how the old mathematicians discovered mathematical facts and tried their experiments.

Check Your Progress

Notes : a) Write your answers in the space given below
b) Compare your answer with the one given at the end of the unit

9. What are the major contributions of Euclid?
……
……

10. List any four important values of History of mathematics which should be known to the mathematics teacher
……
……

1.7 LET US SUM UP

This unit provides the necessary background knowledge on mathematics. The meaning of mathematics are well known to the students with the help of ample number of definitions. Mathematics has

certain unique features which could hardly find in other disciplines. This unit brings out salient features of mathematics such as objectivity, symbolism, abstractness, logical structures, Applicability, Precision and Accuracy.

The history of mathematics has also been elaborated and the contribution of various western and Indian mathematicians have been dealt in detail. This unit also provides an insight into the value and importance of History of mathematics.

1.8 UNIT-END ACTIVITIES

1) Explain the characteristics of mathematics.
2) Discuss the scope of mathematics.
3) Briefly describe the contributions to mathematics by
   a) Bhaskaracharya
   b) Aryabhatta
   c) Brahmagupta
4) Write an essay on history of mathematics.
5) Explain the importance of history of mathematics for a teacher.

1.9ANSWERS TO CHECK YOUR PROGRESS

1. Etymologically the term ‘Mathematics’ is derived from two Greek words: ‘Manthanein’ which means ‘learning and ‘Techne’ which means ‘an art or technique’. The dictionary meaning of mathematics is that ‘it is either the science of number and space or the science of measurement, quantity and magnitude’. Mathematics is thus defined as the science of quantity, measurement and spatial relations.

2. According to Courant and Robbins - "Mathematics is an expression of human mind that reflects the active will, the contemplative reason and the desire for aesthetic perfection. Its basic elements are logic and intuition, analysis and construction, generality and individuality".

   According to new English Dictionary -"Mathematics - in a strict sense - is an abstract science, which investigates deductively, the conclusions implicit in the elementary conceptions of spatial, numerical relations."

3. Objectivity in mathematics is traditionally thought of as one of the more desirable necessities for its own credibility. The basic assumptions are, on the one hand, that any mathematical description of nature must be independent of the wishes, moods, and/or needs of an individual mathematician (or

group of mathematicians), and on a more fundamental basis that nature itself is objective, because it is ruled by laws and not by the concerns or intentions of its inhabitants. The latter is highly critical in that the laws of nature are deemed to be inviolable, quantitative, inexorable, and general; i.e., they should not bend for a purpose.

4. Mathematics uses an unfailing logical structure. The steps in a problem have to be arranged in a logical sequence. Every step or statement to be mathematical must be supported by a proper reason. If we do not do this, then, even the ordinary equations may lead us to fallacies.

5. Even at the elementary stage, mathematics is at the abstract level. For example, number is an abstract concept. When we teach children about numbers we make use of fingers, dolls etc,. We say two and show two balls. Two stands for an abstract concept. Showing balls is a concrete activity. Concretisation helps to develop abstractness.

6. Any mathematical statement should have the following characteristics.

    a) It should make use of conventional signs and symbols.

    b) Every word or symbol used in it should have a definite purpose.

    c) Its language should be clear and unambiguous.

    d) As far as possible, the statement should be simple and brief.

7. The study of mathematics requires the learners to apply the skill acquired to new situations. The knowledge and its application, wherever possible, should be related to daily life situations. Concepts and principles become more functional and meaningful only when they are related to actual practical applications. Such a practice will make the learning of mathematics more meaningful and significant.

8. Mathematics is known as an exact science because of its precision. It is perhaps the only subject which can claim certainty of results. In mathematics the results are either right or wrong, accepted or rejected. There is no midway possible between right and wrong. Mathematics can decide whether or not its conclusions are right.

9. Euclid's major contributions are:

    i. Euclid's second theorem - Prime numbers are infinite.

ii. He thought about the three current problems of time - Dividing an angle into three equal parts, Making double of a cube and obtaining square from a cube.

iii. Euclidean Algorithm-The greatest common divisor of two numbers.

10. The important four values of History of mathematics which should be known to the mathematics teacher

- The teacher can better introduce many mathematical topics in the class by discussing their history.
- It saves the student from attacking an unsolved problem by the same method which has led other mathematicians to failure.
- If the teacher uses his knowledge of the history of mathematics in the classroom, the students form a good impression of his study and knowledge. This helps him in commanding respect.
- The teacher's narration of stories and events, mentioned occasionally, bring about a healthy change in the monotony of the classroom work.

1.10 SUGGESTED READINGS

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UNIT – II
AIMS AND OBJECTIVES

Structure

2.1 INTRODUCTION

While starting to teach a particular subject it is essential to know why we are going to teach that subject. The process of teaching can be kept on right lines only with the help of clear-cut aims. Aimlessness in teaching would result in the wastage of time, energy and other resources.

What should be the aims of teaching mathematics in our schools? The answer requires the knowledge of all the advantages that can be drawn from the teaching of mathematics. These aims will be based on the educational values of the subject. Aims and values are interrelated and interdependent. Aims help in the realization on the values possessed by a subject. This unit gives the clear-cut about aims and values of teaching of mathematics. At the end of this unit we are going to discuss Bloom's Taxonomy of Educational objectives and its three important domains.

2.2 OBJECTIVES

At the end of the unit, the student-teachers will be able to

- Know the importance of aims of teaching mathematics.
- Realize the practical, disciplinary, cultural and social aims of teaching mathematics
- Understand the values of teaching mathematics.
- Bring out the difference between aims and objectives.
- Understand the importance of stating instructional objectives.
- Explain briefly Blooms Taxonomy of educational objectives.

2.3 AIMS OF TEACHING MATHEMATICS

Mathematics is taught in schools because it is useful, disciplines the mind and is beautiful. Every student will have to pick up mathematical knowledge and skills since it cater to the individual, societal and national needs.

While starting to teach a particular subject it is essential to know why we are going to teach that subject. Until we have clear-cut aims of teaching a subject, we would not be able to proceed on the right track. Aimlessness makes the work uninteresting and results in the wastage of time, energy and other material resources both on the part of the teacher and the taught. Therefore, we must have some definite aims of teaching a subject before starting its actual teaching.

The aims give directions for the educational process. The courses offered for study, the curricular and co-curricular activities designed, and the learning experiences provided in the classes are all directed towards the realisation of their aims. Every subject / course included in the curriculum has distinct and unique aims. The aims of teaching mathematics and sciences will be distinctly different from those of teaching languages and social sciences.

Aims of teaching Mathematics give answer to the question “why is Mathematics taught?’ When Mathematics is taught and learnt by the pupils, it is expected that evidence of learning should be available in their behaviour. They are called terminal behaviour. When aims are required to this level of specificity, they become objectives. Objectives signify what the subject of Mathematics is trying to produce in pupils when it is taught.

Aims are expression of strategy, while objectives are technical in character. Objectives specify the “terminal behaviours” of the pupils. Objectives give an account of what the pupil is able to do at the end of a

course of study in terms of remembering, thinking and understanding with respect to the certain subject matter areas.

The entire system of education is geared towards the realisation of certain national goals. The goals of education may be usefully divided into the utilitarian, social, disciplinary and cultural goals; so also the aims of teaching mathematics.

Aims of teaching mathematics can be classified under the following heads.

- Practical Aims
- Disciplinary Aims
- Cultural Aims and
- Social Aims

2.3.1 Practical Aim

The following are the practical aims of teaching mathematics.

- To enable the student to make use of the learning in Mathematics in their day to day life.
- To have clear ideas of number and a comprehension of the way the number is applied to measure of all varieties, but most particularly to those physical concepts he meets with most frequently, length, volume, weight, area, temperature, speed and acceleration.
- Able to apply his knowledge of mathematics to a wide range of problems that continually occur in his everyday life.
- To understand the concepts of ratio and scale drawing and read, interpret graphs, diagrams and tables especially those relating to statistical evidences.
- Able to use correctly, accurately and with understanding the four fundamental operations of addition, subtraction, multiplication and division as applied to both number measurement and check both his own and other peoples calculations by appropriate approximation.
- To provide the basis of mathematical skills and processes which will be needed for vocational purposes.
- To enable the learner to acquire and develop mathematical skills and attitude to meet the demands of (i) daily life (ii) future mathematical work and (iii) work in the related fields of knowledge.

These are the utilitarian or practical goals for every citizen in the world today. The aim is to help the student to lead a rich life with a wide

variety of interests and activities, social, cultural, intellectual, recreational, and creative; mathematics can contribute every one of these directions.

2.3.2 Disciplinary Aim

The teaching of mathematics intends to realise the following disciplinary aims.

- To develop the intellectual powers and discipline the minds of the learners.
- To help the learner in the intelligent use of reasoning power.
- To develop constructive imagination and inventive faculties.
- To develop the character through systematic and orderly habits.
- To help the learner to be original and creative in thinking.
- To help the individual to become self-reliant and independent.

According to Locke, "Mathematics is a way to settle in the mind a habit of reasoning." It trains or disciplines the mind. Due to its very nature, it possesses a real disciplinary value. It is exact, true and to the point knowledge, and therefore creates a discipline in the mind. Its truths are definite and exact. The learner has to argue the correctness or incorrectness of a statement. It taught in the right sense, it develops reasoning and thinking powers more and demands less from memory. The student come to realize that thinking makes him a successful student of all the subjects. Its study results in the development of power rather than the acquisition of knowledge and knowledge also comes as a natural consequence or by-product. Reasoning in mathematics possesses certain characteristics which are suitable for the training of the learners mind. If properly emphasized and streamlined, these characteristics are likely to develop the corresponding habits in the learner. Here ensues a discussion of those characteristics and their influence.

i. **Characteristic of simplicity**: There is a vast scope for simple reasoning in this subject. It teaches that definite facts are always expressed in a simple language and definite facts are always easily understandable. So if you want to be understood, you must express yourself in a definite or simple way. Moreover, one can easily follow a gradation going from simple to complex. The teacher advances by degrees to harder and harder portions. The procedure when practiced for a pretty long time becomes a habit.

ii. **Characteristic of accuracy**: Without accuracy there is no chance of progress and credit in mathematics. Accurate reasoning, thinking and judgment are essential for its study. It is in the nature of this subject that it cannot be learnt through

vagueness of thought and argument. In other subjects, it may sometimes be possible for the student to hide his ignorance by beating about the bush, but such tricks never play in mathematics. Accuracy, exactness and precision compose the beauty of mathematics. The student learns the value and appreciation of accuracy and adopts it as a principle of life. He learns to influence and command others by his accuracy.

iii. **Characteristic of certainty of results**: There is no place for subjectivity and proposal equation in mathematics. The answer is either right or wrong. Subjectivity of difference of opinion between the teacher and the taught. The student can verify his result by reverser process. It is possible for the child to remove his difficulties by self-effort and to be sure of the removal. The success of personal effort is a source of pleasure for him. He develops faith in self-effort which is the secret of success in life. He inculcates the habit of being certain about his achievement.

iv. **Characteristic of originality**: Most work in mathematics demands original thinking. Reproduction and cramming of ideas of others is not very much appreciated. In other subjects, of course, ideas of others occupy a prominent place and have to be grasped by the student. Therefore he can safely depend on memory in other subjects; but without original thinking and intelligent reasoning there cannot be satisfactory progress in mathematics. When he has a new or a different mathematical problem, it is only his originality which keeps him going. The discovery or establishment of a new formula is also his original work. This practice in originality enables the child to face new problems and situations with confidence in his future career.

v. **Characteristic of similarity to the reasoning of life**: Clear and exact thinking is as important in daily life as in mathematical study. Before starting with the solution of a problem, the student has to grasp the whole meaning. Similarly in daily life, while understanding a task, one must have firm grip on the situation. This habit of thinking will get transferred to the problems of daily life also.

vi. **Characteristic of verification of results**: Results can be easily verified. As already pointed out, this gives a sense of achievement, confidence and pleasure. This verification of results is also likely to inculcate the habit of self-criticism and self-evaluation. After making any attempt in life, the child would like to satisfy himself about its success or failure.

2.3.3 Cultural Aim

The cultural aims can be summarised as follows.

- To enable the learner to appreciate the part played by mathematics in theculture of the past and that it continues to play in the present world.
- To enable the student to appreciate the role played by mathematics inpreserving and transmitting our cultural traditions.
- To enable him to appreciate various cultural arts like drawing, design making, painting, poetry, music, sculpture and architecture.
- To provide through mathematical ideas, aesthetic and intellectual enjoyment and satisfaction and to give an opportunity for creative expression.
- To develop in the individual an aesthetic awareness of mathematical shapes and patterns in nature as well as the products of our civilization.

Hence the cultural aim is to help the student to explore the creative fields such as art, architecture and music apart from making them aware of the strength and virtues of the culture they have inherited.

2.3.4 Social Aim

The following are the social aims of teaching mathematics:

- To develop in the individual an awareness of the mathematical principles and operations which will enable the individual to understand and participate in the general social and economic life of his community.
- To help the pupil acquire social and moral values to lead a fruitful life in the society.
- To help the pupil in the formation of social laws and social order needed for social harmony.
- To provide the pupils scientific and technological knowledge necessary for adjusting to the rapidly changing society and social life.
- To help the learner to appreciate how mathematics contributes to his understanding of natural phenomena.
- To help the pupil interpret social and economic phenomena.

Thus the social goals are to make the students understand how mathematics methods namely scientific, intuitive, deductive and inventive are used to investigate, interpret and to make decisions in human affairs and also how it contributes to his understanding of natural phenomena. The scientific method is that in which one seeks to discover

order, pattern and relations not only in the sets or numbers in series, quantities or measures but in the natural world as well. Teachers should arrange for activities and situations to make it possible for students to discover or rediscover the relations for themselves. The intuitive method is one by which advances are made step by step by a flash of insight, and a sudden illumination of a concept brings understanding of a difficult problem situation. The results of ones insight or intuition are linked logically and thus instruction is closely related to deductive reasoning, the other method inventive method emphasizes the need for providing opportunities to students to make their own investigations and find solutions. Besides the mathematical value of discussion, co-operation, learning the importance of organization and corporate endeavour.

Some other aims are,

Aesthetical and Recreational Aim:

To develop their aesthetic abilities, meet their varying interests and help them in the task of utilization of their leisure time.

Moral Aim:

To help them in imbibing essential moral virtues.

Vocational Aim:

To prepare for the future vocation or occupation.

Pre-preparational Aim:

To help in the study of other subjects and future learning in Mathematics.

Inter-disciplinary Aim:

To give them insight to recognize relationships between different branches and topics of Mathematics.

Self-learning Aim:

To help them in becoming self-dependent for mastering new topics and problems of Mathematics.

Check Your Progress

Notes : a) Write your answers in the space given below

b) Compare your answer with the one given at the end of the unit

1. Write a short note on aims of teaching mathematics.

   ……

   ……

2. List out any four practical aims of teaching mathematics.

   ……

   ……

3. How does Mathematics discipline the minds of the students?

   ……

   ……

2.4 VALUES OF TEACHING OF MATHEMATICS

The aims and values are interrelated and interdependent things. One aims at a thing because one values it or by aiming at a thing one would be able to realise its values. Therefore, aims help in the realisation of values or drawing of advantages while the knowledge of the advantages or values of a subject helps in setting the aims to get all the essential advantages.

This makes us to conclude that the knowledge of the advantages or, values of teaching Mathematics may help us a lot in setting the aims of teaching Mathematics in our schools. In other words it may help us in realising what we should expect from our Mathematics teaching.

2.4.1 Practical Values

The use of Mathematics in everyday life is immense. From birth to death, from morning to night, everyone faces quantitative situations in every part of the environment.

Language and numbers are the tools with which we manipulate the world of ours. Of these two, we have to give greater importance to number as a primary instrument.

The practical value of teaching Mathematics is to enable every individual capable of making superior adjustment with quantitative environment. Even a labourer has to calculate and buy things. There are a large number of growing occupations like tailoring, carpentry, house building, business management etc. which require Mathematical results. Every citizen has opportunities to use mathematical facts beyond the barest elements of Arithmetic. The whole commercial system, industry and manufactures are based upon Mathematics. The atmosphere we live in is charged with Mathematics,

The accuracy and exactness of a science is determined to a large extent by the amount of Mathematics used in it. Even social science like Economics, Psychology and Geography make much use of Mathematics for clarifying their concepts. The gigantic works of construction like bridges, dams, towers, works of architecture, building ships, planes, sending satellites and rockets into the space have all become possible only because of this quantitative science.

Mathematics is closely connected with life and natural phenomenon. In the mighty task of understanding the environment and adjusting oneself interestingly with the forces created by the World of Nature. Mathematics is indispensable. In transacting the ordinary affairs

of life, one comes across ideas, explained in Arithmetic like, ratio and proportion, average areas, volumes, interest, stocks and. shares, insurance etc. The graphs, equations and formula have become very common. Fundamental concepts of Algebra and simple geometrical relationship occur frequently in the life of every individual. Thus mathematical facts and skills have become so important in the life of every individual that he will not be able to understand his environment without a proper grounding in Mathematics.

Every phenomenon in life has two aspects; the quantitative and qualitative. The quantitative aspects can be studied only through a Mathematical treatment. To illustrate the knowledge that stem is produced, when water is heated is a quantitative knowledge. This of no use unless, it is subjected to a quantitative analysis like how much pressure can be produced by a particular quantity of steam, what should be the thickness of the tubes to withstand a particular amount of pressure etc.

Physical and Natural Sciences use all the aspects of Mathematics from logarithm and formulae to integration-of functions. This is an age of technology. Hence almost all the professions now-a-days require a Mathematical knowledge;

Thus widespread are the applications of Mathematics and enormous are its practical values Mathematics is fundamental to all - human experience, knowledge and activities. The present school syllabus in Mathematics rightly lays emphasis on the practical aim. Practical activities are included-wherever possible. Today, the method of teaching is not based upon memorisation but it involves the use of multi-sensory aids, correlation, concretisation and rationalization. By these techniques the teacher can make his teaching effective from the practical point of view.

2.4.2 Disciplinary Values

Many abilities are developed as a result of the study of Mathematics. The pupils develop a capacity to generalise from particular statements. They are able to express statements accurately and precisely. The observe fact, critically. Perceive relationships and distinguish relevant between and irrelevant facts.

The pupils develop the ability to concentrate on problems. They are able to keep in memory important facts and processes. They develop the ability to imagine abstract ideas. They are able to think functionally. They develop the capacity to systematically organize and interpret the given data.

They reach correct conclusions by accurate and logical reasoning. They are able to do original thinking. The study of Mathematics develops a power to use symbols and express ideas symbolically. It

develops perseverance, self-confidence, self-reliance, and reverence for truth. The study of Mathematics is thus useful to discipline the mind and create valuable habits and attitudes in the learner.

2.4.3 Cultural Values

All development around us is gift of mathematics. The understanding of world in which man lives, of the civilization to which he belongs and of the culture of which he is proud requires understanding of scientific and social principles, the understanding of which depends on mathematical principle. It has been truly said that "Mathematics is the mirror of civilization."

Modern Civilization stands on the foundation of applied Mathematics. It is expected that a man of culture should have some ideas about the role played by Mathematics in various fields of life.

Culture stands for two aspects. Culture may mean the hall mark of civilization. It may also mean the way of life led by people. Under both these aspects Mathematics has some important values.

The essence of culture consists of a) broad mindedness b) depth of understanding c) appreciation of art and beauty d) reverence for truth e) striving towards the good, noble and pure. Mathematical truths, perfect and symmetrical figures, the purity of numbers, beautiful number patterns and geometrical designs-have all cultural values.

Mathematics is a source of pleasure to many. The Mathematical investigations contained in plant growth, building of spider's web honey comb etc. have been precisely brought out as result or recreational pursuits.

Pythagoras the famous Mathematician and Philosopher has remarked, Gymnastics, Music and Mathematics, should form part of the important studies for enlightenment. By the first-people are strengthened, by the second they are purified by the third they are made ready for the society of men and Gods. Plato considers Mathematics as a fine tool for awakening the soul and stirring up the sleepy and the uninstructed spirit. Summer feels that cultural values and practical values are by no means opposed to each other.

"Learn Arithmetic", the child was advised, for it will teach you to analyse a situation; to come to a decision, to check your thinking and its result, to perceive relationships to concentrate to be accurate and to perform written work systematically and neatly.

2.4.4 Social Values

We know that individuals live in society and there cannot be any society without individuals. Hence practical values are also social values.

We cannot demarcate where practical values end and where social values begin.

Mathematics plays an important role in the organization and maintenance of our social structure. Society is the result of the inter-relations of individuals. It consists of big and small groups and there are sub-groups within each group. Mathematics enables us to understand the inter-relations of individuals and the possibilities of various groups.

Society is a phenomena of balancing and counter-balancing of various social forces. Mathematics helps in creating a social order in this phenomena. It regulates the functioning of society in many ways. Social conditions like justice, fairplay, healthy competition, symmetry, harmony, etc. have often to be described in mathematical terms for the purpose of clarity.

For smooth transactions, exchange, trade, business and bargaining, mathematics becomes a useful tool. It has its own role to play in the development of means of communication. It has helped in knitting the vast society into a family. When the dealings between individuals are given a mathematical touch, it leads to social progress, prosperity and welfare.

History of mathematics reveals that whenever a society gave due weightage to the knowledge of mathematics, it made a tremendous progress. When mathematics makes its contribution in the advancement of science and technology, society draws huge benefits. Its contribution is evident in the fields of atomic energy, space research, space travel, and man-made satellites.

The various scientific advances and the rapid growth of industry have linked all the people of world and made them interdependent. Social institutions like banking, accountancy, insurance navigation, railway and transports, postal and telegraphy, engineering and architecture, trade and commerce have all become closely connected with the lives individuals.

In everyone of the phases of a) origin b) establishment c) maintenance
d) use of social institutions, mathematics is widely used. The success of an individual in a society depends on how well he is able to become a part of the society and what contributions he can make towards the progress of the society depends on how well he can be benefited by the society. Today, our social existence is totally governed by the scientific and technological knowledge which can only be attained by the study of mathematics. An individual can lead a normal social life only when he has the elementary mathematical knowledge to comply with the societal demands. Moreover, the values acquired through learning mathematics such as tolerance, open-mindedness, objectively, co-operation, honesty,

truthfulness, and willpower will help the individual to adjust himself and lead a harmonious life in the society. These we call the Social values of the teaching of Mathematics.

Check Your Progress

Notes : a) Write your answers in the space given below

b) Compare your answer with the one given at the end of the unit

4. How aims and values are interrelated and interdependent things?

……

……

5. Write a short note on cultural values of teaching mathematics.

……

……

6. What are the practical use of Mathematics in everyday life?

……

……

2.5 OBJECTIVES OF TEACHING MATHEMATICS

For a teacher it is practically impossible to realize all the aims of mathematics education within the framework of curriculum, for they involve a total programme of education encompassing even out-of-classroom experiences. The part of the aim that can be achieved within an institution (or in a classroom) is an objective. While aims give directions to education, objectives are directed towards the aims.

Table : Difference between Aims and Objectives

Aims	Objectives
• Give directions to the educational system	Directed towards aims
• Long term goals	Short term goals
• Attainment is beyond the scope of the curriculum	Attainable within the educational system-steps towards the realization of the aims
• Broad and general	Specific, precise and observable
• Common for many disciplines	Vary from course to course. Specific for each course

What is an Objective?

An instructional objective is a statement of expected result. It is a description of the learning outcome that the teacher hopes will result from the instruction, whether in a lesson unit or course. It is a statement of what students should be able to do at the end of the learning period that they could not do before hand. Thus the term 'objective' may be defined as:

"An objective is a point or end view of the possible achievement in terms of what a student is able to do when the whole educational system is directed towards educational aims".

Thus an objective is apart of the aim which a school can hope to achieve. Hence an objective is a narrower term when compared to an aim.

An instructional objective is a statement that describes what the pupil will do or be able to do, towards the realization of an educational aim. When a pupil attains an objective, he realizes a part of the broad aim. In other words, an objective is a statement of the terminal behavior expected of the pupils at the conclusion of a period of learning.

Examples:

- The student is able to discriminate among mathematical terms, symbols, concepts, principles, generalizations etc.
- The student is able to explain theorems in mathematics.
- The student is able to classify postulates, axioms and theorems in mathematics.

Need for Stating Objectives

If teaching is planned based on the curriculum alone, then the emphasis is on the teacher's performance. The teacher is more concerned with: "What knowledge can I dictate to the student''? or "How can I cover the syllabus"? When we use objectives that define the outcome of students' learning or expected performance at the end of the course, then the focus of the teaching learning process gets shifted to the students. A teacher can then get answers to such questions as:

"How can the student grow and change through acquiring and using knowledge and information?

How is the student who has finished this course different from one who has not taken it?

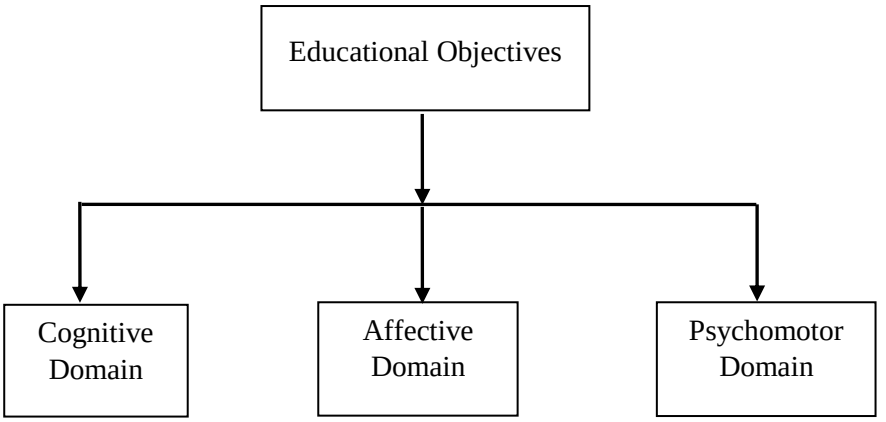
What can the students do with their knowledge after they have finished their course?

What evidence do I have that my *students* actually have reached the objectives set?

The instructional objectives give directions to the teacher in designing her instruction, guide the examiner in selecting suitable evaluation techniques and help the students in knowing what is expected of them after completing the period of learning.

2.5.1 Bloom's Taxonomy of Educational Objectives

Bloom and his associates developed a Taxonomy of Educational Objectives. Taxonomy is a complex scheme for classification of phenomena or ideas. Bloom's taxonomy is an attempt to identify, define, classify and organize a comprehensive range of educational objectives into a compact and measurable structure. Bloom separated the educational objectives relating to the behaviour of individuals under three dimensions. These are called Domains. The classification is shown in the following figure:



Bloom's taxonomy of educational objectives consists of three domains: the cognitive, the affective and the psychomotor. The objectives each domain constitute a hierarchy consisting of major categories and subcategories within the major categories. The taxonomy is hierarchical in the sense that levels increase in difficulty and sophistication and it is cumulative in the sense that each level builds on the ones below.

COGNITIVE DOMAIN

Cognitive Domain includes those objectives which deals with the recall or recognition of knowledge and the development of intellectual abilities and skills. This domain consists of six classes of objectives arranged in a hierarchical order, in increasing levels of difficulty and sophistication.

Judgement	
Synthesis	↑
Analysis	↑

Application	↑
Comprehension	↑
Knowledge	↑

Knowledge forms the basis for all classes of objectives and all other levels of objectives are not possible without knowledge. Similarly if one is able to apply a rule or a principle to a new situation, it can be inferred that he or she has achieved the lower levels of objectives namely knowledge and comprehension. Thus, a higher level objectives are presented in the following table:

Objectives	**Descriptions**
Knowledge	The ability to remember and recall or recognize already learnt information.
Comprehension	The ability to organize and arrange materials mentally without necessarily relating to other material.
Application	The ability to select and apply already learnt rules, procedure and principles to new and unfamiliar situation.
Analysis	The ability to break up a given communication into its constituent elements or parts such that the relative hierarchy of ideas expressed is more explicit.
Synthesis	The ability to draw upon elements from many forces and put them together to produce a new and unique communication.
Judgment (Evaluation)	The ability to make judgment about some value of materials and methods for given purposes judgment can be both quantitative and qualitative.

The action verbs tabulated in the following table are very helpful for a teacher to state the specific objective relating to cognitive domain. A teacher can select appropriate action verb, keeping in mind the objective, the content and the level of achievement of the student.

Objectives	**Action Verbs and Examples**
Knowledge	**Key Words:** defines, describes, knows, labels, lists, matches, names, outlines, recalls, recognises, reproduces, selects, states. **Examples:** Recite previous classroom activity. Quote formula from memory in the classroom. Knows Geometric principles.
Comprehension	**Key Words:**Comprehends, converts, defends, distinguishes, estimates, explains, extends, generalises, gives examples, infers, interprets, paraphrases, predicts, rewrites, summaries,

	translates. **Examples:** Rewrites the principles of test writing. Explain in one's own words the steps for performing a critical problem. Translates a word problem into a symbolic form.
Application	**Key words:** applies, changes, computes, constructs, demonstrates, discovers, manipulates, modifies, operates, predicts, prepares, produces, relates, shows, solves, uses. **Examples:** Use logarithmic table to solve logarithmic problems. Apply laws of statistics to evaluate the reliability of a written test.
Analysis	**Key Words:** analyses, breaks down, compares. Contrasts, diagrams, deconstructs, differentiates, discriminates, distinguishes, identifies, illustrates, infers, outlines, relates, selects, separates. **Examples:** Solving problems using logical induction. Recognise Mathematical fallacies reasoning. Gathers information from a department and selects the required tasks for training.
Synthesis	**Key Words:** categories, combines, compiles, composes, creates, devises, designs, explains, generates, modifies, organises, plans, rearranges, reconstructs, relates, reorganises, revises, rewrites, summarises, tells, writes. **Examples:** Write the process of computer. Design a flow chart to solve a program. Integrates training from several sources to solve a problem. Revises and process to improve the outcome. Identify suitable formula to solve the given problem.
Judgment (Evaluation)	**Key Words:** appraises, compares, concludes, contrasts, criticises, critiques, defends, describes, discriminates, evaluates, explains, interprets, justifies, relates, summarises, supports. **Examples:** Select the most effective solution. Explain the method of drawing triangle. Explain and justify a new budget.

AFFECTIVE DOMAIN

The affective domain addresses interests, attitudes, opinions, appreciations, values and emotional sets. The objectives and their descriptions are presented in the following table:

Objectives	Descriptions
Receiving	Awareness, willingness to hear, selected attention.
Responding	Active participation on the part of the learners.

	Attends and reacts to a particular phenomenon. Learning outcomes may emphasise compliance in responding, willingness to respond or satisfaction in responding (motivation).
Valuing	The worth or value a person attaches to a particular object, phenomenon, or behavior. This ranges from simple acceptance to the more complex state of commitment. Valuing is based on internalisation of a set of specified values, while clues to these values are expressed in the learner's overt behavior and are often identifiable.
Organisation	Organises values into priorities by contrasting different values, resolving conflicts between them, and creating a unique value system. The emphasis is on comparing, relating, and synthesising values.
Characterisation	Characterisation has a value system that controls their behavior. The behavior is pervasive, consistent, predictable, and most importantly, characteristic of the learner. Instructional objectives are concerned with the student's general patterns of adjustment (personal, social, emotional).

In affective domain, characterisation is the highest level of objective and includes the remaining four objectives. If the instruction is intended for accomplishing changes in values and attitudes then the instruction should be geared to progress through the levels listed in as above.

The action verbs tabulated in the following table are very helpful for a teacher to state the specific objective relating to affective domain. A teacher can select appropriate action verb, keeping in mind the objective, the content and the level of achievement of the student.

Objectives	**Action Verbs and Examples**
Receiving	**Key Words:** Asks, chooses, describes, follows, gives, holds, identifies, locates, names, points to, selects, sits, erects, replies, uses. **Examples:** Listen the drawing principles. Listen for and remember the name of newly introduced geometrical figures.
Responding	**Key Words:** answers, assists, aids, complies, conforms, discusses, greets, helps, labels, performs, practices, presents, reads, recites, reports, selects, tells, writes. **Examples:** Participates in class discussions. Conform the solution to the problem. Questions new ideals, concepts, models, etc. In order to fully understand them. Know the

Objectives	Action Verbs and Examples
	formulae and practices them.
Valuing	**Key word:** Completes, demonstrates, differentiates, explains, follows, forms, initiates, invites, joins, justifies, proposes, reads, reports, selects, shares, studies, works. **Examples:** Demonstrates Pythagoras theorem. Differentiate graphical and non-graphical method of problem solving. Shows the ability to solve problems. Proposes a plan to collect statistical data and follows through with commitment. Works continuing with previous classroom activities.
Organisation	**Key words:** adheres, alters, arranges, combines, compares, completes, defends, explains, formulates, generalises, identifies, integrates, modifies, orders, organises, prepares, relates, synthesises. **Examples:** Arrange the given numbers in the ascending order and descending order. Relates the given problem with known problem. Explains the role of systematic planning in solving problems. Organise the splitted parts of the geometrical figure and combine them in the meaningful form. Integrates the previously learned principles to solve the given problem. Prioritises time effectively to solve the simple and complicated problems.
Characterization	**Key words:** Acts, discriminates, displays, influences, listens, modifies, performs, practices, proposes, qualifies, questions, revises, serves, solves, verifies. **Examples:** Shows self-reliance when working independently cooperates in group activities (displays teamwork). Uses an objective approach in problem solving. Displays home works on a daily basis. Revises the results with the given answer at the end of the lesson. Values the classroom work on the basis of performance.

PSYCHOMOTOR DOMAIN

In the psychomotor domain the focus is on the development of motor skill. The Psychomotor domain includes those objectives that are concerned with the development of manual and motor skills. The suggested areas include manipulative skills required in business training, industrial technology and performance areas in science, art and music. The educational objectives and their descriptions are presented in the following table:

Objectives	Descriptions
Imitate	Observe a skill and attempt to repeat it, or see a finished product and attempt to replicate it while attending to an

	exemplar.
Manipulate	Perform the skill or produce the product in a recognisable fashion by following general instructions rather than observation.
Precision	Independently perform the skill or produce the product, with accuracy, proportion, and exactness.
Articulation	Modify the skill or produce the product to fit new situations; combine more than one skill in sequence with harmony and consistency.
Naturalisation	Completion of one or more skills with ease and making the skill automatic with limited physical or mental exertion.

The action verbs tabulated in the following table are very helpful for a teacher to state the specific objective relating to psychomotor domain. A teacher can select appropriate action verb, keeping in mind the objective, the content and the level of achievement of the student.

Objectives	Action Verbs and Examples
Imitate	**Key Words:** Attempt, copy duplicate, imitate, mimic **Examples:** Copying the blackboard work of geometry.
Manipulate	**Key Words:** Complete, follow, play, perform produce. **Examples:** Creating work on one's own, after learning the concept, or reading about it.
Precision	**Key Words:** Achieve automatically. Excel expertly. Perform masterfully. **Examples:** Working and reworking of graphical problems, so it will be "just right"
Articulation	**Key Words:** Adapt, alter, customize, originate. **Examples:** Producing a video that involves number systems, logrithm, Algebra, Geometry, etc.
Naturalization	**Key Word:** Naturally, perfectly **Examples:** Naturally counting the numbers. Perfectly draw the figure in the notebook as it is in the blackboard.

Check Your Progress

Notes : a) Write your answers in the space given below

b) Compare your answer with the one given at the end of the unit

7. What is an objective?

……

……

8. What are the six major classes of Cognitive Domain?

……

……

2.6 LET US SUM UP

This unit provides a deep insight into the aims and objectives of teaching of mathematics.The term aims of teaching mathematics stands for the goals, targets or broader purposes that may be fulfilled by the teaching of mathematics in the general scheme of education. The objectives are those short-term, immediate goals or purposes that may be achieved within the specified classroom situation. While aims give directions to education objectives are directed towards the aims.

Every teacher of mathematics needs to be informed and convinced about the educational values of his subject. This unit brings out its educational soundness to the student teachers to adopt effective methods, devices and illustrative materials.

Bloom's taxonomy of educational objectives consists of three domains the cognitive, the affective and the psychomotor. The objectives in each domain constitute a hierarchy consisting of major categories and subcategories.

2.7 UNIT-END ACTIVITIES

1) Discuss briefly the aims of teaching mathematics.
2) What are the main values of teaching mathematics? Discuss in detail.
3) Discuss the relationship between aims and objectives of teaching mathematics.
4) What do you understand by the term Taxonomy of Instructional objectives? Discuss the Bloom's Taxonomy of Instructional objectives related to the cognitive domain of the behaviour.
5) Discuss in detail the Taxonomy given by Bloom and his associates for the instructional objectives in affective domain of the behaviour.

2.8 ANSWERS TO CHECK YOUR PROGRESS

1. The aims give directions for the educational process. The courses offered for study, the curricular and co-curricular activities designed, and the learning experiences provided in the classes are all directed towards the realisation of their aims. Every subject / course included in the curriculum has distinct and unique aims. The aims of teaching mathematics and sciences will be distinctly different from those of teaching languages and social sciences.

Aims of teaching Mathematics give answer to the question "why is Mathematics taught?'.

2. The following are the practical aims of teaching mathematics.
    - To enable the student to make use of the learning in Mathematics in their day to day life.
    - To have clear ideas of number and a comprehension of the way the number is applied to measure of all varieties, but most particularly to those physical concepts he meets with most frequently, length, volume, weight, area, temperature, speed and acceleration.
    - Able to apply his knowledge of mathematics to a wide range of problems that continually occur in his everyday life.
    - To understand the concepts of ratio and scale drawing and read, interpret graphs, diagrams and tables especially those relating to statistical evidences.
3. According to Locke, "Mathematics is a way to settle in the mind a habit of reasoning." It trains or disciplines the mind. Due to its very nature, it possesses a real disciplinary value. It is exact, true and to the point knowledge, and therefore creates a discipline in the mind. Its truths are definite and exact. The learner has to argue the correctness or incorrectness of a statement. It taught in the right sense, it develops reasoning and thinking powers more and demands less from memory. The student come to realize that thinking makes him a successful student of all the subjects.
4. The aims and values are interrelated and interdependent things. One aims at a thing because one values it or by aiming at a thing one would be able to realise its values. Therefore, aims help in the realisation of values or drawing of advantages while the knowledge of the advantages or values of a subject helps in setting the aims to get all the essential advantages. This makes us to conclude that the knowledge of the advantages or, values of teaching Mathematics may help us a lot in setting the aims of teaching Mathematics in our schools. In other words it may help us in realising what we should expect from our Mathematics teaching.
5. Modern Civilization stands on the foundation of applied Mathematics. It is expected that a man of culture should have some ideas about the role played by Mathematics in various fields of life. Culture stands for two aspects. Culture may mean the hall mark of civilization. It may also mean the way of life led by people. Under both these aspects Mathematics has some

important values. The essence of culture consists of a) broad mindedness b) depth of understanding c) appreciation of art and beauty d) reverence for truth e) striving towards the good, noble and pure. Mathematical truths, perfect and symmetrical figures, the purity of numbers, beautiful number patterns and geometrical designs-have all cultural values.

6. The use of Mathematics in everyday life is immense. From birth to death, from morning to night, everyone faces quantitative situations in every part of the environment. Language and numbers are the tools with which we manipulate the world of ours. Of these two, we have to give greater importance to number as a primary instrument. The practical value of teaching Mathematics is to enable every individual capable of making superior adjustment with quantitative environment. Even a labourer has to calculate and buy things. There are a large number of growing occupations like tailoring, carpentry, house building, business management etc. which require Mathematical results. Every citizen has opportunities to use mathematical facts beyond the barest elements of Arithmetic. The whole commercial system, industry and manufactures are based upon Mathematics.
7. An objective is a point or end view of the possible achievement in terms of what a student is able to do when the whole educational system is directed towards educational aims.
8. Six major classes of Cognitive Domain are Judgement, Synthesis, Analysis, Application, Comprehension, Knowledge

2.9 SUGGESTED READINGS

- Agarwal, S.M. (1994). *Teaching of Modern Mathematics*; DhanpatRai& Sons, New Delhi.
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NOTES

UNIT – III

MATHEMATICS CURRICULUM

Structure

3.1 INTRODUCTION

The mathematics curriculum forms the basis for the entire mathematics education. It is the pivot on which the whole process of teaching –learning revolves. It provides the necessary insight to the mathematics teacher in the selection of the learning activities, teaching methods, learning resources and evaluation techniques. It helps the students in getting trained in skills necessary for his individual to social development, for selecting vocations in life, studying other subjects and pursuing a higher education in mathematics. A good curriculum provides experiences which are best suited to the age of the learner, the emotional, physical and intellectual maturity of the learner of his previous experiences and learning.

The teacher has to play a major role in curriculum development. He has to come forward with his suggestions so that curriculum may be made most effective of purposeful. Assimilation of the entire background of theory of curriculum construction is thus important for him.

This unit discusses various principles of curriculum and some modern trends in curriculum development.

3.2 OBJECTIVES

At the end of the unit, the student-teachers will be able to

- Define curriculum
- Understand the principles of curriculum
- Appreciate the changes in the modern curriculum
- Get the clear knowledge about the characteristics of modern mathematics curriculum.

3.3 MATHEMATICS CURRICULUM

Mathematics curriculum forms an important part of the school curriculum. The Indian Education Commission (1966) emphasised the importance of mathematics in the school curriculum in view of the importance of quantification and the advent of automation and cybernetics in the scientific and industrial revolution. Mathematics curriculum comprises all those systematically planned activities carried out by the student individually/ in group with /or without the supervision of the teacher inside or outside the classroom/ school with the definite purpose of realising the aims and objectives of teaching mathematics.

3.3.1 Meaning of curriculum

The word curriculum is derived from the Latin word ‘currere’ which means ‘run’. Thus curriculum means a course to be run for reaching a certain goal or destination. Thus the traditional definition of curriculum is a course of study or training leading to a product or education. Teaching- learning process does not operate in a vacuum. Certain planned experiences have to be provided in school so that optimum human development according to the needs of a particular country is possible. Thus the term curriculum in recent years has come to mean all the planned activities and experiences available to the students under the direction of the school. Curriculum is dynamic and changes according to the needs of the pupils and society.

3.3.2 Definition of Curriculum

Curriculum has been defined differently by many authors and over the years the focus is being shifted from ‘course of study’ to ‘learning activities and experiences’. According to Alberty, A. and Alberty, E., (1959) curriculum is the sum total of student activities which the school sponsors for the purpose of achieving its objectives. H. Robert Beck and W. Walter Cook define curriculum as” The sum of the educational experiences that children have in school”. Cunningham says “Curriculum is a tool in the hands of the artist (teacher) to mould his

material (the pupil) in accordance with his ideals in his studio (the school)".

Derek Rowntree in 'A Dictionary of Education' (1981) has defined curriculum as "The total structure of ideas and activities developed by an educational institution to meet the needs of students and to achieve educational aims". Thus it is clear that curriculum covers not only the courses of study but also covers all the wider areas of individual and group life. It also encompasses all the meaningful and desirable activities outside the school, provided that these are planned, organised and used educationally. A good curriculum in fact is the sum total of good learning experiences that the pupils have in order to achieve the goals of education which determine the direction of these experiences.

3.3.3. Need for Mathematics Curriculum

The mathematics curriculum seeks to answer the following.

- What learning experiences are most appropriate for the attainment of the objectives of teaching mathematics?
- What kind of subject matter (syllabus) would provide the best learning experiences to the students to realise the objectives?
- What kind of mathematical skills are to be developed among the students to lead a fruitful life in the modem and technologically advanced society?
- What kind of resources are necessary for effective handling of instruction in the mathematics classroom?
- What contributions can the learning of mathematics make towards the individual and national development?
- What kind of mathematical knowledge should be provided to the students which are useful for him in his daily life?
- What kind of mathematical skills would help the students in taking up a fairly good number of vocations?
- What is the type of mathematical knowledge necessary for the study of other subjects and for a higher education in mathematics?
- What are the best methods of teaching and the most appropriate techniques of evaluation best suited to the student?

Thus the mathematics curriculum forms the basis for the entire mathematics education. It is the pivot on which the whole process of teaching- learning revolves. It provides the necessary insight to the mathematics teacher in the selection of the learning activities, teaching methods, learning resources and evaluation techniques. It helps the

students in getting trained in skills necessary for his individual and social development, for selecting vocations in life, studying other subjects and pursuing a higher education in mathematics. A good curriculum provides experiences which are best suited to the age of the learner, the emotional, physical and intellectual maturity of the learner and his previous experiences and learning.

3.3.4 Major Objectives of the Mathematics Curriculum

The mathematics curriculum aims at the following objectives.

- Proficiency in fundamental mathematical skills.
- Comprehension of basic mathematical concepts.
- Appreciation of significant meanings.
- Development of desirable attitudes.
- Efficiency in making sound mathematical applications.
- Confidence in making intelligent and independent interpretation.

Check Your Progress

Notes : a) Write your answers in the space given below
b) Compare your answer with the one given at the end of the unit

1. Write a short note on mathematics curriculum.
   ……
   ……
2. Give any two definitions of curriculum.
   ……
   ……
3. List out any four objectives of mathematics curriculum.
   ……
   ……

3.4 CURRICULUM DEVELOPMENT

Though the definition of curriculum has been changing according to the social changes and society's expectation for school, the process of curriculum development has remained unaltered. It is a cyclic process involving the following stages.

Analysis : What are the educational goals of the institution?

Design : What are the educational experiences to be provided to achieve these goals?

Implementation : How can these educational experiences be effectively organised?

Evaluation : How effective are the educational experiences in attaining the goals?

Through these stages of curriculum development the curriculum planners set goals, plan experiences, select content and assess outcomes of a school programme.

Curriculum development is fundamentally a plan of structuring the environment to coordinate in an orderly manner the elements of time, space, materials, equipment and personnel. The basic cycle -analysis, design, implementation and evaluation, guides the curriculum improvement process, regardless of focus or operation.

Thus in developing a curriculum, three choices must be made; of syllabus content, of classroom experiences or pedagogical 'style' and of evaluative (or assessment) techniques. Effective curriculum planning provides evidences that the teacher in deciding the 'what' of the lesson, has also considered the 'how' and given thought to asking 'what will constitute evidence of attainments'? These three decisions then lock together into experiences which provide a learner with the structures necessary to make effective classroom experience.

The curriculum planning involves two major stages:

i) Curriculum construction and

ii) Curriculum organisation

3.4.1 Principles of Curriculum Construction

There are certain basic principles of curriculum planning which should form the basis for the construction of a good mathematics curriculum. They are as follows:

- **Principle of Child-centeredness**: The curriculum shall be based on the present needs and capabilities of the children. The curriculum should help in developing initiative, cooperation and social, responsibility Curriculum should be related to everyday life: The curriculum should provide sufficient opportunities to the students to relate what they learn in classrooms among the children. This implies that the curriculum should meet the physical, intellectual, emotional and social needs of the pupils.
- **Curriculum should provide a fullness of experiences for children**: The curriculum should help the children in living a wholesome and self-fulfilling life. The curriculum should be responsive to the fast changing realities of life.

- **Curriculum should be dynamic and not static**: Curriculum should reflect growth and movement of life. The curriculum should accommodate the latest developments in mathematics, science and information technology.
- With the daily life experiences. The problems and theories that form a part of the mathematics curriculum should be real and should help in solving everyday life problems.
- It must take into account the economic aspect of life of the people to whom an educational institution belongs: The curriculum should provide adequate opportunities for the children to become economically self-reliant through participation in SUPW (Socially Useful and Productive Work) and through vocationalisation. Moreover the curriculum should prepare the child in taking up a good vocation if it happens to discontinue its education before completing high school or higher secondary school. The future of the student in a technological age has to be given due consideration while constructing the mathematics curriculum.
- **Curriculum should be real and rationalistic**: The curriculum should be real and should facilitate rational and original thinking.
- **Curriculum should lay emphasis on learning to live rather than living to learn**: Learning for the sake of learning is not to be encouraged. Curriculum should contain such information and experiences which can be assimilated and put into use for vital life situations.
- **Curriculum should help in preserving and transmitting our cultural traditions**: Curriculum should contain such activities that help in preserving and spreading the culture of our nation.
- **Curriculum should be flexible and elastic**: Curriculum should provide a variety of activities keeping in view the requirements of the students of different communities, regions, (rural or urban) and socio-economic strata.
- **Curriculum should emphasise attitudes rather than acquisition of knowledge**: The emphasis in curriculum activities should be more on development of right attitudes among the students than on mere acquisition of knowledge.
- **The curriculum should be well integrated**: The mathematics curriculum should be in conformity with the curriculum of other subjects. There should be continuity and

coherence within mathematics and with respect to other subjects as well. The learning of mathematics should help in viewing knowledge as a whole and not as segments of information

- **The curriculum should provide both for uniformity and variety**: The curriculum should provide uniformity in terms of the content and objectives and variety in terms of the activities and experiences provided to the students on the basis of their academic, social, intellectual and environmental requirements.
- **The curriculum should be useful to the students**: While selecting topics due consideration should be given as to how useful the topics are in
    i) daily life
    ii) the study of other subjects
    iii) higher education
    iv) selecting a good number of a vocations
    v) appreciating the role played by mathematics in the development of our culture and civilization.

3.4.2 Principles of Curriculum Organisation

The topics or contents selected for inclusion in the curriculum should have a proper order and arrangement as to facilitate the teaching and learning of a sequence subject like Mathematics. What topics or contents should be reserved for the curriculum of one stage or the other or what should be the scope and limit of the content in a particular class or stage should be properly decided in developing curriculum for a particular class or stage. The following principles need to be considered for this purpose:

1. Principle of Logical and Psychological order.
2. Principle of Activity.
3. Criterion of difficulty.
4. Principle of concentric or Spiral arrangement.
5. Principle of correlation.

1. Principle of Logical and Psychological order

There are two different viewpoints for the organisation of the contents or topics in Mathematics - one Psychological and the other logical. The former advocates the organisation according to the development of the mind of the child, his needs and interests etc. whereas the latter considers, systematic knowledge more important than the child. It demands that the topics should be taught in a logical order depending upon the fundamental processes and modes of thinking. In

this way. logic recommends that volume should be studied after studying all topics related to area, but practically the volume is simpler than many topics in area $i.e.$ area of circle etc. Therefore, psychologically it is improper to teach in the way as logic suggest. On the other hand we cannot do away with logic as the systematic organisation of the subject cannot be sacrificed.

In this way we need to organise the topics and contents in such a way that we may follow psychology and logic at one and the same time. It is not difficult. The point is that we can be logical in a number of ways. Psychology should decide what type of logic is suitable for the students of a certain age and abilities and what kind of topics will be most suitable for the development of such logical thinking.

2. Principle of Activity

Child is active by nature. Research in the field of pedagogy reveals that.child learns through direct experiences more than the indirect experiences. He should be given more opportunity for using concrete things in a junior class in order to learn skills and acquire useful knowledge. Therefore, a topic which gives greater scope of practical work (direct experiences and handling of concrete objects) should appear in lower classes: and it should be allowed to grow gradually into the abstract principles of Mathematics to be studied in successive higher classes.

3. The Criterion of Difficulty

In the organisation of the contents or topics we must try to follow the maxim "from simple to complex." What we put in terms of topics and contents in a curriculum of any class or stage should suit the mental capacity and development of the age group of that class or stage. It means that there should neither be too easy nor too difficult topics in a curriculum. The topics of the beginning classes should' be within the comprehension of the children without involving any complication; as the students go higher and higher on the ladder, the topics may take a more and more difficult form. It should also be considered that what is simple for a Mathematics teacher may not necessarily be simple for the student. Therefore, in every case the difficulty of a topic must be judged from the point of view of the pupils.

4. Principle of Concentric or spiral arrangement

While organising topics or contents of a curriculum it is to be thought that whether we should cover a topic as a whole in a particular grade or should it be spread over to different grades by covering easier portion in the lower grades and the difficult ones in the higher and higher grades. We can come to the conclusion that the latter arrangement which is known as concentric or spiral arrangement is better than the former

topical arrangement. In organising topics, therefore spiral arrangement should be followed in the manner mentioned below.

i) Every topic should be divided into a number of smaller independent units or parts ; (ii) these parts should be graded according to the difficulty and (iii) each should be introduced when it is needed or when the pupils have attained the proper stage of development for its introduction.

Principle of Correlation.

While organising topics or contents of a curriculum, one must be careful to seek the maximum possible correlation of the topics or contents.

i) With the life activities of the pupils,

ii) With the teaching of other subjects of the school curriculum,

iii) With the topics of the other branches of Mathematics,

iv) With the topics or contents of the branch to which they themselves belong,

v) With the particular experience or activities of the Work-experience areas.

In seeking the above five types of correlation the following kinds of information need to be gathered about the students for whom curriculum is being constructed:

i) Social, physical and cultural environment of the students.

ii) Day to day life activities of the students.

iii) Nature of the other subjects taught and the possibility of seeking correlation with them.

iv) Nature of the topics or contents of the different branches of Mathematics being taught in the different grades.

v) Nature of the experiences gained in work-experience areas of, different grades and possibility of seeking correlation with them.

3.5 CHARACTERISTICS OF MODERN MATHEMATICS CURRICULUM

Certain characteristic features of a modern mathematics curriculum may be specified briefly as follows.

- Mathematics course materials should prepare the students for college, but could be used with less talented students if they are given more time.
- New concepts and different points of view which are useful for the students should be there in the mathematics curriculum.
- Changes in the curriculum should help the students in meeting their present needs.
- The curriculum should provide an understanding of mathematics for future change and development.
- The curriculum should provide application of mathematical structures and matric and non-matric relations in geometry.
- The curricular materials should involve experience with and appreciation of abstract concepts, the role of definitions, the development of precise vocabulary and thought and experimentation and proof.
- An emphasis on structure of algebra for a clear understanding of the sound mathematics is essential in the mathematics curriculum.
- The mathematics curriculum should provide experiences to explore the behaviour of numbers and invent new numbers to describe' new situations.
- The mathematics curriculum should be in harmony with the cultural experiences the children have at home and outside the school.
- The mathematics curriculum should be built on the mathematical experiences that the students already have gained.

Check Your Progress

Notes : a) Write your answers in the space given below

b) Compare your answer with the one given at the end of the unit

4. List out any six Principles of Curriculum Construction

……

5. What are the Principles of Curriculum Organization?

……

6. State any six Characteristics of Modern mathematics Curriculum

……

NOTES

3.6 CRITICAL ANALYSIS OR EVALUATION OF A SYLLABUS IN MATHEMATICS

In every subject the required syllabus for a particular class or stage middle, Secondary or higher secondary-is set for rendering help in the realisation of the proposed objectives of teaching the subject at that stage. Since education is a state subject, the State Department or Board of Education formulates its own Syllabus in different subjects for different grades or Stages according to the needs of its people, availability of the local resources and facilities available in its schools. Critical study of a syllabus is carried out to a certain degree for knowing the competency to carry out the task for which it has been set. This critical analysis should be as objective as possible. The answering of related questions about the different aspects of the curriculum may provide help in this direction.

A. Criteria Related to the Objectives of Teaching Mathematics

i) Does the Syllabus indicate the objectives of teaching Mathematics at the particular stage?

ii) Are the listed objectives sufficient and adequate?

iii) Have these objectives been properly classified into specific categories?

iv) Have the objectives been clearly defined in terms of expected behavioural changes?

v) Does the syllabus lay emphasis on the attainment of set objectives?

B. Criteria Related to the Selection and Organisation of the Contents for Topics

i) Have the contents or topics been related to the utilisation criterion?

ii) How far does the subject matter cater in the needs of higher studies in the subject?

iii) Does the syllabus give consideration to the needs, age level, interest and other psychological aspects of the students?

iv) Are the contents or topics well graded?

v) Is the organisation of the contents or topics according to the topical or Spiral method

vi) Does the selection and organisation follow the principle of integration and correlation?

vii) Do the contents or topics lay emphasis on independent problem solving?

viii) Has the syllabus adequate scope for oral, written, drill or home assignment?

C. Criteria Related to the Selection of Learning Experiences and Learning Situations

i) Does the syllabus suggest a list of suitable learning experiences for realisation of proposed objectives?

ii) How far are these suggestions helpful in the realisation of the setobjectives?

iii) Is the time allotted per week for the subject in the Syllabus satisfactory?

iv) Does the Syllabus recognize and cater for the individual differences in children?

v) Does the Syllabus leave scope for independent individual projects or hobbies?

vi) Does the Syllabus suggest variety of teaching aids or learning materials?

vii) Does the Syllabus have any indication regarding teaching methods or plan?

viii) Does the Syllabus have useful suggestions for the authors of the text books in the subject?

ix) Are there any suggestions for the activity, experiences and practical work for the teachers?

x) Does the Syllabus give proper suggestions regarding text books, reference materials and books for further study?

D. Criteria Related to the Suggestions about Evaluation Plan

i) Does the Syllabus discuss the evaluation plan for assessing the students' progress?

ii) Do the methods and techniques suggested in Syllabus fulfil the objectives of evaluation in the subject?

iii) Does it mention the distribution of marks according to the division of the subject into branches?

iv) Does the Syllabus mention qualifying pass marks in the subject and the mode of examination-home, public-at the end of each session?

3.7 LET US SUM UP

This unit equips the student-teachers with adequate knowledge about certain basic principles of curriculum planning which should form the basis for the construction of a good mathematics curriculum.

This unit deals with the curriculum designing in mathematics. A good mathematics curriculum leads to better mathematical learning outcomes in pupils. The principles of curriculum designing in mathematics are used to guide the evolution of the aims and objectives, the structure of the curriculum and the identification of objectives in mathematics.

This unit covers the characteristics of modern mathematics curriculum and critical analysis of mathematics curriculum.

3.8 UNIT-END ACTIVITIES

1) What are principles underlying the construction of mathematics curriculum? Discuss.
2) Describe the principles of curriculum organisation.
3) What points should be borne in mind while evaluating critically a syllabus? Discuss in detail.
4) What changes and improvements you would propose in the existing mathematics curriculum of our schools?
5) Compare different curricula in Mathematics of various streams.

3.9 ANSWERS TO CHECK YOUR PROGRESS

1. Mathematics curriculum forms an important part of the school curriculum. The Indian Education Commission (1966) emphasised the importance of mathematics in the school curriculum in view of the importance of quantification and the advent of automation and cybernetics in the scientific and industrial revolution. Mathematics curriculum comprises all those systematically planned activities carried out by the student individually/ in group with /or without the supervision of the teacher inside or outside the classroom/ school with the definite purpose of realising the aims and objectives of teaching mathematics.
2. According to Alberty, A. and Alberty, E., (1959) curriculum is the sum total of student activities which the school sponsors for the purpose of achieving its objectives.

   Cunningham says “Curriculum is a tool in the hands of the artist (teacher) to mould his material (the pupil) in accordance with his ideals in his studio (the school)”.

3. The mathematics curriculum aims at the following objectives.
   - Proficiency in fundamental mathematical skills.
   - Development of desirable attitudes.
   - Efficiency in making sound mathematical applications.
   - Confidence in making intelligent and independent interpretation.
4. Principles of Curriculum Construction
   - Principle of Child-centeredness
   - Curriculum should provide a fullness of experiences for children
   - Curriculum should be dynamic and not static
   - Curriculum should be related to everyday life
   - Curriculum should be real and rationalistic
   - Curriculum should lay emphasis on learning to live rather than living to learn.
5. Principles of Curriculum Organization
   - Principle of Logical and Psychological order.
   - Principle of Activity.
   - Criterion of difficulty.
   - Principle of concentric or Spiral arrangement.
   - Principle of correlation.
6. Characteristics of Modern Mathematics Curriculum

- New concepts and different points of view which are useful for the students should be there in the mathematics curriculum.
- Changes in the curriculum should help the students in meeting their present needs.
- The curriculum should provide an understanding of mathematics for future change and development.
- The curriculum should provide application of mathematical structures and matric and non-matric relations in geometry.
- The curricular materials should involve experience with and appreciation of abstract concepts, the role of definitions, the development of precise vocabulary and thought and experimentation and proof.
- An emphasis on structure of algebra for a clear understanding of the sound mathematics is essential in the mathematics curriculum.

NOTES

3.10 SUGGESTED READINGS

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UNIT – IV

METHODS AND STRATEGIES - I

Structure

4.1 INTRODUCTION

A Mathematics teacher has a variety of methods and strategies available for use in teaching mathematics. The selection of a suitable method depends upon the objectives of the lesson, needs of the learner and the nature of the content. This unit covers various methods and strategies such as expository and discovery methods, co-operative and collaborative strategies,Lecture Method, Laboratory Method, Heuristic Method and Project Method.

4.2 OBJECTIVES

At the end of the unit, the student-teachers will be able to

- Understand the formation of mathematical concepts
- Acquire skill in teaching mathematics through several teaching methods and strategies
- Select and apply different strategies of teaching mathematics
- Understand the steps involved in project method.

- Understand the merits and demerits of Lecture Method, Laboratory Method, Heuristic Method and Project Method.

4.3 FORMATION OF MATHEMATICAL CONCEPTS

NOTES

A concept is a generalization that helps to organize information into categories. A concept has elements viz. Name, Examples, Attributes rules and etc.

Formation of mathematical concepts consists of the following steps viz. Familiarization, similarity recognition, Reification and Application.

Formation of Mathematical concepts helps students to organize complex notions into simpler and therefore more easily usable forms.

Concept Attainment

How concepts are attained is basic not only for understanding mathematics but also to the way the subject is taught. While research on concept learning provides no well-defined strategy which assumes every student will acquire a desired concept, there are considerable date providing cues to the more fruitful instructional practices. Forming a concept is an individual affair and is therefore influenced by the range of characteristics distinguishing one student from another, his intellectual ability, motivation and teaching conditions. The way a course is organized and the teacher's mode of instruction will also influence a student's attainment of a concept.

A significant factor in developing concepts is the learner's previous experience. Familiarity with a topic is an advantage because it provides a base for incorporating new information. The influence of their knowledge depends upon its stability, structure, clarity, meaningfulness and relevance. When teaching mathematics, concepts related to concrete situations which have substances in direct observation are better acquired than abstract ideas. Students who are aware of how to go about the task of encoding and decoding information are more likely to attain a desired concept than those students without this awareness. Forming a concept is a search in process, deploring and unwieldy collection of facts and finding out similarities and difference of organizational properties and for a meaningful integration of the same.

In the process of abstracting a common property from a mass of information, the student looks for logical relationships, constructs and tests them by identifying which features characterize most of the data to the exclusion of other information. The process is one of discriminating, categorizing and evaluating in a logical and meaningful manner, always striving get a better arrangement of the data. The concept emerges gradually as information is progressively recognized. There are times when it may seem to emerge rapidly as in a moment of insightful

enlightenment or sudden recognition, but this is usually a final response and occurs after one has spent considerable item consistently searching for identities.

The Concept Attainment Model

The concept attainment model developed by Joyce and Weil is based on Bruner's theory of concept attainment. The three variations of this models the reception oriented model, the selection oriented concept attainment model and unorganized material model.

The Reception Oriented Concept Attainment Model

This model is more direct in teaching students the elements of a concept and their use in concept attainment.

PHASE I:

Presentation of Data and Identification of the Concept

- ❖ Teacher presents labeled examples
- ❖ Students compare attributes of positive and negative examples
- ❖ Students generate and test hypotheses
- ❖ Students state a definition according to the essential attributes

PHASE II

Testing Attainment of the Concept

- ❖ Students identify additional unlabeled examples as yes or no
- ❖ Teacher confirms students hypotheses
- ❖ Teacher names the concept
- ❖ Teacher restates definition according to essential attributes
- ❖ Students generate examples

PHASE III

Analysis of the Thinking Strategies

- ❖ Students describe thoughts
- ❖ Students discuss role of hypotheses and attributes
- ❖ Students discuss type and number of hypotheses
- ❖ Teacher evaluates the strategies

In the first phase, the teacher presents the positive and negative examples in a sequence. This data may be in the form of pictures, diagrams, events or illustrations. The pupils are told that there is only one idea common in all the positive examples and that they have to compare and justify the attributes and from some hypothesis about the concept. When the pupils have analyzed the examples and forms the

hypotheses the teacher ask the students to arrive at definition according to the essential attributes.

In phase two, to test student's attainment of the concept, the teacher presents unlabeled examples. The students categories them as positive or negative. The teacher probes for reasons and confirms their hypotheses.

When the teacher knows that the students have attained the concept, the teacher names the concept as the students are not familiar with the name of the concept. Only when the students have already attained the concept, the teacher may ask the students to name the concept. To test the attainment of the concept further the teachers asks the pupils to generate examples and labeled them as positive or negative instances of the concept.

In the third phase of the model, the teacher analyses the thinking strategies, employed by the students. The students report their pattern of thinking, their hypotheses, focus attributes or concepts and the process of hypothesizing with all its trials and errors.

Selection Oriented Concept Attainment Model

PHASEI:

Presentation of Data and Identification of the Concept

- ❖ Teachers presents unlabeled examples
- ❖ Students generate and test hypotheses
- ❖ Students inquire which examples are positive based on the first positive instance given by the teacher.

PHASE II

Testing Attainment of the Concept

- ❖ Students identify additional unlabeled examples
- ❖ Students generate examples
- ❖ Teacher confirms hypotheses, names concept and restates definition according to the essential attributes

PHASE III

Analysis of the Thinking Strategies

- ❖ Students describe thoughts
- ❖ Students discuss role of hypotheses and attributes
- ❖ Students discuss type and number of hypotheses
- ❖ Teacher evaluates the strategies

The procedure under selection strategy begins with the presentation of several instances representing the various combinations

of attributes of a concept. The teacher then draws the attention of the student to some of the examples resented before him which illustrate the concept in the mind. The teacher introduces by showing them and instance that illustrates the positive examples of the concept. The pupil task is to select examples form those presented to them, verify them, one at a time against the first positive example and label them as positive or negative. The pupils may select the examples in any order they choose but one at a time. The pupils thus generate hypotheses, test them and arrive at the definition of the concept. The second and third phase of the selection model are the same as that of the reception model. Only in the third phase while analyzing the thinking strategies, the teacher keeps in the mind selection thinking strategies which are different from the reception theory strategies.

The Unorganized Material Model

The procedure for analyzing concepts in the unorganized material involves.

- ❖ Locating the concepts
- ❖ Identifying the attributes used
- ❖ Discussing the adequacy and appropriateness of attributes
- ❖ Comparing of examples using the same concept.

Check Your Progress

Notes : a) Write your answers in the space given below
b) Compare your answer with the one given at the end of the unit

1. Write a short note on Formation of mathematical concepts.
   ……
   ……
2. List out the phases in Reception Oriented Concept Attainment Model.
   ……
   ……

4.4 EXPOSITORY TEACHING METHOD

In an expository classroom, students take notes and gauge success by tests and other measurements. Using an expository teaching method, a teacher uses his knowledge to explain or tell about the subject or concept being learned. The teacher lectures, and the students listen and take notes as needed. A good expository teacher will have a firm grasp of the subject and an organized approach to revealing it in a logical sequence for the students. While this method might encourage a static

classroom environment, it doesn't have to be that way. A good teacher will encourage questioning and stimulate discussion.

This method is teacher controlled and information-centred and in this method the teacher works as a sole resource in classroom instruction. In this method the students are provided with readymade information by the teacher. The teacher goes ahead with the subject matter at his own speed.

Merits of Expository Teaching Method

- Using this method, a large amount of subject matter can be presented within a short time and the prescribed syllabus can be covered easily.
- It is an easy, concise and attractive method. Using this method the teacher feels safe and secure.
- It can be used for a large number of students.
- Using this method it is quite easy to impart factual information and historical anecdotes.
- Using this method the teacher can easily maintain the logical sequence of the subject matter by planning his lecture in advance.
- This method gives the students as well as the teacher a sense of satisfaction and achievement.
- This method trains good listeners who are able to concentrate on subject matter for a long duration.

Demerits of Expository Teaching Method

- The Expository Teaching method is lengthy and time-consuming.
- In this method students become passive recipients of information as their involvement in classroom interaction is negligible.
- Receiving information is not mathematics learning and hence it does not enhance mathematical ability of the students.
- In this method there is no way to ensure the students' concentration and understanding of the subject matter presented to them.
- In this method ideas are presented so rapidly that it is not possible for all students, especially the weaker ones to catch up with the teacher's presentation.

- Inability to understand one essential point may make the rest of the lecture unintelligible.
- It does not provide for corrective feedback and remedial help to slow learners.
- It does not call for the development of mental faculties such as power of observation, reasoning, critical thinking, independent thinking and so on.
- It does not provide for individual differences and individual needs.
- This method does not help in developing problem-solving skills.
- Using this method results in totally neglecting the experimental side of learning.
- It is a teacher-oriented method and it is likely that students profit very little due to lack of maturity of thought and many other psychological reasons.

Expository Teaching method is most suitable under certain situations such as:

- When information can be given more effectively through Expository Teaching method than any other method with respect to the understanding, retention and economy of time.
- While introducing a new concept, a topic or a unit.
- While summarising and reviewing certain concepts.
- When the general enthusiasm or any effective emotion is to be raised.
- While initiating a discussion.
- When the teacher has the desired available data, not easily obtained by the class.

4.5 DISCOVERY TEACHING METHOD

This method as the name implies is a method by which the pupil discovers things for himself. The pupil is put in the position of a pioneer and finds his way along the path of knowledge as done by those who first discovered the facts, principles and laws which are now known to all. Therefore, it is a method in which the student discovers facts and information, relationships and principles for himself. It is a method based on the principle of learning by doing. The Discovery Teaching method, thus basically provides training in scientific method; acquisition of knowledge is a secondary consideration. In this method, students are

trained to collect data, interpret data and arrive at solutions by rejecting superfluous statements.

The primary objectives of the method are to

- inculcate in the student the habit of enquiry and research
- enable him to listen, to observe, to ask and to discover.
- make the pupils more exact, observant and thoughtful.
- lay solid foundation for future self-learning.
- inculcate the spirit of scientific enquiry.

Essential Conditions for the Discovery Teaching method

The following conditions should be kept in mind while following the Discovery Teaching method.

- Freedom of action to the students.
- Providing responsive environment
- Minimum help from the teacher
- Timely guidance from the teacher when it is absolutely essential
- Asking constructive questions
- Availability of necessary supplementary material like library books, Internet resources and so on.
- Encouragement to continue learning through discovery teaching method
- Providing well-graded problems (experiments) to match the level and ability of the student.

Merits of Discovery Teaching method

- It helps in the realisation of aims of teaching mathematics.
- It develops the habit of enquiry and investigation among students.
- It helps in the development of mathematical sense and reasoning.
- It lays stress on individual practical work, careful observation and independent thinking which make the student self-reliant.
- It imparts knowledge of scientific method of thinking and inculcates scientific attitude among the students.
- It is a psychologically sound system of learning as it is based on the principle of learning by doing.

- It provides for individual differences as each student can work at his own pace.
- It helps in real understanding and enduring mastery over the subject matter.
- It enhances the problem-solving skills of the student as each student rediscovers the mathematical principles and rules.
- It helps in the development of social skills as the students have to cooperate with one another.
- It helps in establishing cordial relationship between teacher and the taught as the teacher is in close contact with every student in the class.

Demerits of Discovery Teaching method

- It is a lengthy, time-consuming method and hence it is difficult to cover the prescribed syllabus in time.
- This method is feasible only with a highly resourceful teacher and a small class. Both these requirements are difficult to attain practically.
- Suitable textbooks are not available.
- Many mathematical truths cannot be discovered by students themselves.
- It is not suitable for beginners as the students need enough guidance at early stages to have a proper foundation in the subject.
- In this method, the stress is more on skills than on acquisition of knowledge. This could lead to an imbalance in learning, as knowledge is of secondary consideration
- Not all students can cope with this type of learning.
- This method requires special preparation on the part of the teacher. He has to put in more labour and effort.
- Evaluation of learning through this method can be quite tedious.
- Most of the teachers lack sprit of discovery which poses a major threat to implementing this method.

Check Your Progress

Notes : a) Write your answers in the space given below

b) Compare your answer with the one given at the end of he unit

3. What is expository method?

……

……

4. State primary objectives of discovery teaching method

……

……

4.6 CO-OPERATIVE AND COLLABORATIVE STRATEGIES

Collaborative Learning

Collaborative learning is a situation in which two or more people learn or attempt to learn something together. Unlike individual learning, people engaged in collaborative learning capitalize on one another's resources and skills (asking one another for information, evaluating one another's ideas, monitoring one another's work, etc.). More specifically, collaborative learning is based on the model that knowledge can be created within a population where members actively interact by sharing experiences and take on asymmetry roles. Collaborative learning refers to methodologies and environments in which learners engage in a common task where each individual depends on and is accountable to each other.

Collaborative learning is commonly illustrated when groups of students work together to search for understanding, meaning, or solutions or to create an artifact or product of their learning. Further, collaborative learning redefines traditional student-teacher relationship in the classroom which results in controversy over whether this paradigm is more beneficial than harmful. Collaborative learning activities can include collaborative writing, group projects, joint problem solving, debates, study teams, and other activities. The approach is closely related to cooperative learning. Collaborative learning is a method of teaching and learning in which students team together to explore a significant question or create a meaningful project.

Cooperative learning

Cooperative learning is a generic term that is used to describe an instructional arrangement for teaching academic and collaborative skills to small, heterogeneous groups of students. Cooperative learning is deemed highly desirable because of its tendency to reduce peer competition and isolation, and to promote academic achievement and positive interrelationships. A benefit of cooperative learning, therefore, is to provide students with learning disabilities (LD), who have math disabilities and social interaction difficulties, an instructional arrangement that fosters the application and practice of mathematics and collaborative skills within a natural setting (i.e., group activity). Thus, cooperative learning has been used extensively to promote mathematics achievement of students both with and without LD.

According to the National Council of Teachers of Mathematics, learning environments should be created that promote active learning and teaching; classroom discourse; and individual, small-group, and whole-group learning. Cooperative learning is one example of an instructional arrangement that can be used to foster active student learning, which is an important dimension of mathematics learning and highly endorsed by math educators and researchers. Students can be given tasks to discuss, problem solve, and accomplish.

Cooperative learning activities can be used to supplement textbook instruction by providing students with opportunities to practice newly introduced or to review skills and concepts. Teachers can use cooperative learning activities to help students make connections between the concrete and abstract level of instruction through peer interactions and carefully designed activities. Finally, cooperative learning can be used to promote classroom discourse and oral language development.

4.7 LECTURE METHOD

This method is teacher controlled and information-centered and in this method the teacher works as a sole resource in classroom instruction.

Lecture Method refers to the teaching procedure to clarify or explain to the students some ideas that have been presented or created as a problem. In this method the students are provided with readymade information by the teacher. The teacher goes ahead with the subject matter at his own speed.

GUIDELINES FOR PREPARING A LECTURE

To make the lecture method effective, the teacher has to follow certain guidelines while preparing the lecture.

- Lectures should be carefully and systematically planned as the teacher plans his lesson.
- Objectives should be kept in mind while preparing the lecture.
- All pertinent and relative illustrative devices and demonstrations must be carefully incorporated in the lecture.
- The lecture should be carefully outlined.
- The appreciative experiences of the class and general principles of induction and deduction must be taken into account.
- The lecture should exhibit the thorough knowledge of the subject matter, its organization, development, interpretation and application.
- Where the lecture involves narration or description, they should be coordinated with day-to-day life experiences of the students.

- Student participation should be encouraged by asking questions, clarifying the doubts and reviewing the key points at frequent intervals.

EXAMPLE

NOTES

Topic : "Profit and Loss" .

The teacher goes on telling and explaining "Well students, profit and loss is always to be calculated on the cost price, because the cost price is our investment in the bargain. If you invest less and earn more you gain; therefore gain is to be calculated by subtracting cost price from the selling price. When you invest more and earn less, you lose; therefore loss is to be calculated by subtracting selling price from the cost price, so on and so forth".

Applicability of Lecture Method

Lecture method is not quite suitable for high school classes because school students do not have the intellectual maturity that enables them to assimilate immediately and adequately any lengthy one-sided presentation of unfamiliar subject matter. Moreover, this method is not suitable for teaching mathematics as it does not check the development of mental faculties such as power of observation, inductive reasoning, reflective thinking, critical thinking, and problem solving which as essential for leaning mathematics. Lecture method does not provide for application principles and formulae through drill and practice. However, if carefully planned, lecture method can be effectively used for teaching mathematics at higher levels.

Lecture method is most suitable under certain situations such as;

- When information can be given more effectively through lecture method than any other method with respect to the understanding, retention and economy of time.
- While introducing a new concept, a topic or a unit.
- While summarizing and reviewing certain concepts.
- When the general enthusiasm or any effective emotion is to be raised.
- While initiating a discussion.
- When the teacher has the desire available date, not easily obtained by the class.

Merits of Lecture Method

Lecture method has the following merits.

- It is an easy, concise and attractive method. Using this method the teacher feels safe and secure.

- Using this method, a large amount of subject matter can be present within a short time and the prescribed syllabus can be covered easily.
- It can be used for a large number of students.
- Using this method it is quite easy to impart factual information and historical anecdotes.
- Using this method the teacher can easily maintain the logical sequence of the subject matter by planning his lecture in advance.
- This method gives the students as well as the teacher a sense of satisfaction and achievement.
- Lecture method trains good listeners who are able to concentrate on subject matter for a long duration.

Demerits of Lecture Method

The demerits of lecture method are as under:

- The lecture method is lengthy and time-consuming.
- In this method students become passive recipients of information as their involvement in classroom interaction negligible.
- Due to long duration of lecture, students' attention I likely to wander.
- Receiving information is not mathematics learning and hence it does not enhance mathematical ability of the students.
- In this method there is no way to ensure the students' concentration and understanding of the subject matter presented to them.
- In this method ideas are presented so rapidly that it is not possible for all students, especially the weaker ones to catch up with the teacher's presentation.
- Inability to understand one essential point may make the rest of the lecture unintelligible.
- It does not provide for corrective feedback and remedial help to slow learners.
- It does not call for the development of mental faculties such as power of observation, reasoning, critical thinking, independent thinking and so on.
- It does not provide for individual differences and individual needs.
- This method does not help in developing problem-solving skills.

- Using this method results in totally neglecting the experimental side of learning.
- It is a teacher-oriented method and it is likely that students profit very little due to lack of maturity of thought and many other psychological reasons.

Check Your Progress

Notes : a) Write your answers in the space given below
b) Compare your answer with the one given at the end of the unit

5. What is Collaborative learning?

……
……

6. Mention any two demerits of lecture method.

……
……

4.8 LABORATORY METHOD

In this Laboratory Method students are required to do some experiments or carry out certain activities in order to verify the validity of a mathematical generalization, a law or a statement. It is the experimental portion of the inductive method or the practical form of the heuristic method. Therefore, in this method one proceeds from concrete to abstract. This approach makes the learning process more interesting, lively and meaningful.

Laboratory Method is a procedure of stimulating the activities of the students and to encourage them to make discoveries.

According to J.W.A. Young "a room specially filled with drawing instruments, suitable tables and desks, good blackboards and the apparatus necessary to perform the experiment of the course is really essential for the best success of the laboratory method".

EXAMPLE

For the determining the ratio between the circumference and the diameter of a circle, laboratory method will involve the following steps:

(i) Students may be asked to draw circles with a diameter of 7cm on their cardboard pieces and thus to cut out such circular figures.

(ii) Ask them to measure the circumference of these circular figures either by

a) Measuring the length of the thread tied around the figure or

b) Measuring the distance traveled by the circular figure on rolling it down (without Sliding) on a piece of paper while it makes one complete revolution.

(iii) Ask them to compare the measured circumference with the diameter.

(iv) Ask them to repeat the same above experiment with the circular pieces of 3.5 cm, 14 cm, or 21 cm. diameter and note the deduced results.

(v) Now help the students to think inductively and establish the fact that $\frac{\text{Area of a circle}}{\text{diameter}}=22/7=\pi$, (where π, a Greek letter denotes a constant value 22/7)

Laboratory method is easily applicable for the calculation of the area of the figures like rectangle, square, triangle and circle etc. as may be seen from the following example:

Area of rectangle:

(i) Ask the students to take the rectangle piece of a cardboard and measure its length and breadth. Let them be 8 cm and 5 cm.

(ii) Ask them to divide the length into 8 equal parts and breadth into 5 equal parts. They may, again be asked to draw lines from these points parallel to length as well as breadth. The cardboard piece in this way will be divided into $8\times5=40$ similar square pieces.

(iii) Ask to measure the area of anyone of these square pieces. It will be a unit square i.e. 1 sq.cm. In this way the area of the 40 square pieces will be equal to 40 sq.cm.

(iv) The above experiment may then be repeated with the help of other rectangular cardboard pieces of different dimensions and the students may be persuaded to conclude inductively that area of a rectangle= Its Length × its breadth.

SOME SAMPLE TOPICS FOR LABORATORY METHOD

Listed below are some topics from high school mathematics which could be treated through laboratory method.

1. Derivation of the formulae for the,

   i) Circumference of a circle, area of a circle.

   ii) Area of square, rectangle, parallelogram, and trapezium.

iii) Area of triangle, right angled triangle, isosceles right-angled triangle.

iv) Total surface area of cone, cylinder.

v) Volume of a sphere.

vi) Volume of a cone.

2. Expansion of identities such as.

$(a-b)^2$;$(a+b+c)^2$,$(a+b)^2$ (a - b); (x +a) (x -b);$(a-b)^3$etc.

3. Verification of

i) Properties of certain geometrical figures like parallelogram, rhombus etc.

ii) Angle sum property in a triangle

iii) Transversal properties.

iv) Properties of vertically opposite angles formed by intersecting lines.

v) Congruency postulates.

vi) Theorems relating to triangles and circles.

Merits of Laboratory Method

- It is based on the psychological laws of learning: law of exercise and law of effect.
- It is based on the principle of learning by doing.
- It stimulates the interest of the students to work with concrete material
- It provides an opportunity for the students to verify the validity of the mathematical rules through their application.
- Knowledge and skills acquired through experiments help in better understanding and longer retention.
- It provides for individual differences and best suited for average and below average students for through understanding of abstract concepts.
- It promotes self-confidence and self-reliance and a sense of achievement among the students.
- It provides opportunities for social interaction and cooperation among the students.
- It develops in the child a habit of scientific enquiry and investigation.

Demerits of Laboratory Method

- Laboratory method does not contribute much towards the mental development of the students.
- It is an expensive method in terms of time, equipment, laboratory facilities and number of skilled and able teachers.
- Only very few topics in mathematics can be taught through this method and hence it has limited applicability.
- It is too much to expect the students to work independently and discover and verify mathematical facts like a mathematician.
- It is not suitable for larger classes as the teacher has to give individual attention to each student.
- It is suitable only for lower classes.
- There is a dearth of textbooks written on the lines of laboratory method.

4.9 HEURISTIC METHOD

The term 'heuristic' is derived from the Greek work 'which means 'I discover'. This method was advocated by Professor H.E. Armstrong of the City and Guilds Institute, London, who felt that by placing the student in the position of a discoverer, he would learn much more than being merely told about things.

In the words of Ryburn, W.M., "Heuristic method as the name implies is a method by which pupil discovers things for himself. The pupil is put in the position of a pioneer and finds his way along the path of knowledge as done by those who first discovered the facts, principles and laws which are now known to all". Therefore, it is a method in which the student discovers facts and information, relationships and principles for himself.

THE PRIMARY OBJECTIVES OF THE METHOD

- inculcate in the student the habit of enquiry and research
- enable him to listen, to observe, to ask and to discover.
- make the pupils more exact, observant and thoughtful.
- lay solid foundation for future self-learning.
- inculcate the spirit of scientific enquiry.

ESSENTIAL CONDITIONS FOR HEURISTIC LEARNING

The following conditions should be kept in mind while following heuristic method.

- Freedom of action to the students.

- Providing responsive environment.
- Minimum help from the teacher.
- Timely guidance from the teacher when it is absolutely essential
- Asking constructive questions
- Availability of necessary supplementary material like library books, Internet resources and so on.
- Encouragement to continue learning through heuristic method.
- Providing well-graded problems (experiments) to match the level and ability of the student.

Example -1

Problem:

Ravi bought a cow for Rs.500/- . Later on at his own need he sold it to Kannan for Rs.400/- Try to find out his loss percentage .

Procedure:

The students will be asked to study this problem carefully. Then, the teacher will lead them to the solution of the problem through the suitable heuristic questions as illustrated below:

1. What have you to find out in this problem?

   Loss percentage.

2. How will you find out the loss percentage?

   First of all net loss will be calculated. Then will lead us to compute loss percentage.

3. Try to find out the net loss in this problem?

   Students will discover that net loss to Ravi is of Rs.100/-

4. How will you compute loss percentage now?

   Students, on the basis of their previous knowledge know that profit and loss always incurs on the cost price. Hence they can readily respond that this Rs.100 loss has incurred on Rs.500/- They also know that percentage is always calculated on 100 and hence they can very well proceed to solve the problem.

In this way the students may be persuaded to solve such more questions and then they may be helped to generalize the following formulae related to computation of loss or gain percentage.

Loss percentage $= \frac{Netloss \times 100}{CostPrice}$

Profit percentage $= \frac{Netprofit \times 100}{CostPrice}$

Merits of Heuristic Method

- This is a psychological method.
- It develops self-confidence, self-reliance and scientific attitude.
- It develops ability of observation and spirit of enquiry to solve the problems.
- This method makes them exact and brings them closer to truth.
- Contemplation and awakening increases in the children
- The knowledge obtained by this method is more stable.
- Pupil gets the opportunity to develop the mental and thinking powers.
- This method is based on the psychological principles. Learning by doing takes place in this method.
- This method is based on the principle of activity.
- Individual attention of the teacher is possible and the relation between the teacher and learner becomes more intimate.

Demerits of Heuristic Method

- This method is feasible only with a highly resourceful teacher and a small class. Both these requirements are difficult to attain practically.
- It is a lengthy, time-consuming method and hence it is difficult to cover the prescribed syllabus in time.
- Suitable textbooks are not available.
- Many mathematical truths cannot be discovered by students themselves.
- It is not suitable for beginners as the students need enough guidance at early stages to have a proper foundation in the subject.
- In this method, the stress is more on skills than on acquisition of knowledge. This could lead to an imbalance in learning, as knowledge is of secondary consideration
- Not all students can cope with this type of learning.
- This method requires special preparation on the part of the teacher. He has to put in more labour and effort.
- Evaluation of learning through heuristic method can be quite tedious.
- Most of the teachers lack heuristic sprit which poses a major threat to implementing heuristic method.

Check Your Progress

Notes : a) Write your answers in the space given below

b) Compare your answer with the one given at the end of the unit

7. Mention any two merits of laboratory method.

……

4.10 PROJECT METHOD

Project Method is of American origin and is an outcome of Dewey's philosophy of pragmatism. Project method was advocated by Kilpatrick, an American educationist. This method is based on pragmatic philosophy. This method consists chiefly of building a comprehensive unit around an activity which may be carried on in the school or outside. It involves a variety of activities. In this method all the students work co-operatively.

Definitions of Project

1. **According to Kilpatrick-**"A project is a whole-hearted purposeful activity proceeding in a social environment."
2. **According to Stevenson-**"A project is a problematic act carried to completion in its natural setting."
3. **According to Ballard-**"A project is a bit of real life that has been imported into school."

It is clear from the above definition that a project is a purposeful and problematic activity which is achieved in natural, real and social environment. In this method the problem is presented in a practical and real sense.

STEPS INVOLVED IN PROJECT METHOD

- ❖ Providing a situation
- ❖ Selecting of the project.
- ❖ Planning of the project
- ❖ Executing the project.
- ❖ Evaluating the project.
- ❖ Recording.

CRITERIA OF A GOOF PROJECT

A good project can be assessed using the following criteria.

- The project should be purposeful, useful, and practically applicable to the daily life of the students, with clear, well defined objectives.
- The project should help in providing useful and meaningful learning experiences to each member of the group.
- The project should be within the reach of the students in accordance with their interest and ability levels.
- The project should be feasible in terms of the availability of human and material resources and time limit.
- The level of complexity o f the project should match the ability level of the students.
- The learning activities of the project should be life-like, purposeful and natural.

ROLE OF THE TEACHER

The teacher should assume the following role while following project method.

- Guide students in selecting the project according to their interest, aptitude and ability.
- Help students in planning and allotting activities to each member according to the nature of abilities.
- Help in creating a friendly and democratic atmosphere in the classroom promoting co-operation and harmony.
- Be available to the students and willing to help as and when it is necessary.
- Supervise and check whether the project is running in time as planned.
- Suggest extra resources, if necessary, for the successful execution of the project.
- Check the records maintained by the students.
- Help in the periodic assessment of the progress of the project.

SOME PROJECTS FOR MATHEMATICS

A few projects suitable for high school mathematics are listed below.

- Running a cooperative bank in the school.
- Running a stationery stores in the school.
- Laying out a school garden.
- Laying a road

- Planning and estimating the construction of a house
- Planning for an annual camp.
- Comparison of expenses incurred for a journey using different modes of transport.
- Maintaining shop and studying about gain or loss.
- Running a School Bank.
- Maintaining height-weight records.
- Average consumption of water per head in town.
- Mathematics in kitchen.

Example:

Title: collecting rates of number of post-cards, letters, inland letters, envelopes, international letters.

Step 1: Teacher takes students to Post Office.

Step 2: Different aspects that will be dealt by students are like:

History of Post Office, Total Number of departments in Post-Office, work of different Departments, Cost of different types of letters. Students choose project which is linked to Their actual requirement.

Step 3: students group will be entrusted with various tasks of project, according to their ability, Interest and capacity.

Step 4: students start with the work. Collection of data through different sources, materializing the things, framing hypothesis, checking, re-checking, putting forth ideas, preparing statistical data and tabulation, will be tasks carried out by students.

Step 5: After collecting the entire information rates of different types of letters etc., completed work is evaluated.

Step 6: Recording of all information, typed material or floppies are maintained for future reference.

In this project, students gain learning experience

(1) Number sense is developed

(2) Accounts concerning all types of purchase, sales, income and expenditure are obtained.

(3) Knowledge of different communication modes.

(4) Information of Post Office.

(5) Historical background of Post office.

(6) Moral values are learnt.

(7) Social bond is developed.

MERITS OF PROJECT METHOD

- It is based on sound psychological principles and laws of teaching.
- It provides scope for independent work and individual development.
- It promotes habits of critical thinking and encourages the students to adopt problem
- It provides for individual differences as the students can select the activity and work as their own pace.
- It promotes social interaction, inculcates spirit of co-operation and exchanges of experiences among the students.
- It encourages practical applications of the subject, making the subject functional and meaningful to the learner.
- It provides opportunities for children to acquire a lot of skills – observation, references, interpretation and so on.
- In this the children are active participants in the learning task.
- It develops self-confidence and self-discipline among the students.
- It upholds the dignity of labour.
- It widens the mental horizon of the students
- It makes the learning more interesting and facilitates better understanding of the subject matter as the learning is related to reality and the world around him.

Demerits of Project Method

- The project method is uneconomical in terms of time and is not possible to fit into the regular timetable.
- It does not provide any training in mathematical thinking and reasoning.
- The learning is incomplete and uniform learning or balanced learning is not possible for all students as each student performs a different activity.
- Textbooks and instructional materials are hardly available.
- For the success of this method the teachers should be exceptionally resourceful and gifted and knowledgeable.
- It is an expensive method as it makes use of a lot of resources which are not immediately available in the school.
- Syllabus cannot be completed on time using this method.

- Teaching is disorganised.

Check Your Progress

Notes : a) Write your answers in the space given below

b) Compare your answer with the one given at the end of the unit

9. What is the role of teacher in project method?

……

……

10. Mention any four criteria of a good project.

……

……

4.11 LET US SUM UP

Formation of Mathematical concepts helps students to organize complex notions into simpler and therefore more easily usable forms. Forming a concept is a search in process, deploring and unwieldy collection of facts and finding out similarities and difference of organizational properties and for a meaningful integration of the same.

In an expository classroom, students take notes and gauge success by tests and other measurements. Using an expository teaching method, a teacher uses his knowledge to explain or tell about the subject or concept being learned. The teacher lectures, and the students listen and take notes as needed.The Discovery Teaching method is a method in which the student discovers facts and information, relationships and principles for himself. It is a method based on the principle of learning by doing.

In this unit, a variety of methods of teaching mathematics were discussed in detail. Teaching method is a systematic way in which a teacher uses to transfer or receive or share information. This unit emphasizes the usefulness of teaching method depends on the nature of the content, instructional objectives and needs of the learner. If you want to be a successful and effective teacher you must know how to use each method Effective. You must also understand the Advantages and Disadvantages of the method you are applying. This will help a teacher to overcome the short comes that associated with those methods. Teacher must use alternate number of methods to meet the demand of every student in the class. This is a property of effective and efficient teacher.

The meaning, definition, steps, examples, merits and demerits of important methods of teaching mathematics have been dealt in detail. This unit also provides an insight into the value and importance of methods of teaching mathematics.

4.12 UNIT-END ACTIVITIES

1) Discuss briefly the formation of mathematical concepts.
2) Explain the uses of Expository Teaching method.
3) Describe about co-operative and collaborative strategies.
4) Write the advantages of Lecture method.
5) Write an essay on heuristic method.
6) Explain the importance of laboratory method in teaching of mathematics.
7) Explain in detail – project method.

4.13 ANSWERS TO CHECK YOUR PROGRESS

1. A concept is a generalization that helps to organize information into categories. A concept has elements viz. Name, Examples, Attributes rules and etc. Formation of mathematical concepts consists of the following steps viz. Familiarization, similarity recognition, Reification and Application. Formation of Mathematical concepts helps students to organize complex notions into simpler and therefore more easily usable forms.
2. PHASE I: Presentation of Data and Identification of the Concept

   PHASE II : Testing Attainment of the Concept

   PHASE III : Analysis of the Thinking Strategies
3. Expository method is teacher controlled and information-centred and in this method the teacher works as a sole resource in classroom instruction. In this method the students are provided with readymade information by the teacher. The teacher goes ahead with the subject matter at his own speed.
4. The primary objectives of discovery method are to
   - inculcate in the student the habit of enquiry and research
   - enable him to listen, to observe, to ask and to discover.
   - make the pupils more exact, observant and thoughtful.
   - lay solid foundation for future self-learning.
   - inculcate the spirit of scientific enquiry.
5. Collaborative learning is a situation in which two or more people learn or attempt to learn something together.
6. The demerits of lecture method are:
   - The lecture method is lengthy and time-consuming.

- In this method students become passive recipients of information as their involvement in classroom interaction negligible.

7. Merits of Laboratory Method
   - It is based on the psychological laws of learning: law of exercise and law of effect.
   - It is based on the principle of learning by doing.
8. The term 'heuristic' is derived from the Greek work 'which means 'I discover'. This method was advocated by Professor H.E. Armstrong of the City and Guilds Institute, London, who felt that by placing the student in the position of a discoverer, he would learn much more than being merely told about things.
9. The teacher should assume the following role while following project method.
   a) Guide students in selecting the project according to their interest, aptitude and ability.
   b) Help students in planning and allotting activities to each member according to the nature of abilities.
   c) Help in creating a friendly and democratic atmosphere in the classroom promoting co-operation and harmony.
   d) Be available to the students and willing to help as and when it is necessary.
10. A good project can be assessed using the following criteria.
   a) The project should be purposeful, useful, and practically applicable to the daily life of the students, with clear, well defined objectives.
   b) The project should help in providing useful and meaningful learning experiences to each member of the group.
   c) The project should be within the reach of the students in accordance with their interest and ability levels.
   d) The project should be feasible in terms of the availability of human and material resources and time limit.

4.14 SUGGESTED READINGS

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UNIT – V
METHODS AND STRATEGIES - II

Structure

5.1 INTRODUCTION

This unit provides adequate knowledge about how a teacher could reduce the diversity and to create the equality among heterogeneous students in a classroom.

Dyscalculia is the inability to understand mathematical concepts and this impacts across the whole of a student's mathematical learning. It is our ambition that every student with Special Educational Needs reaches their full potential in school, and can make a successful transition to adulthood and the world of further and higher education, training or work. To promote the welfare and interests of such students and to improve the support they receive, this unit provides practical ideas and advice on how, you as teachers, can raise the levels of achievement and ignite a mathematical imagination for such pupils in your care.

Active learning methodology makes the students as intelligent participants in knowledge creation. The format used for mathematics teaching in Active learning Methodology is TIGER FORMAT. This unit gives a clear picture of how to useTIGER method in the classroom effectively and also this unit deals with merits and demerits of Analytic and synthetic methods.

5.2 OBJECTIVES

At the end of the unit, the student-teachers will be able to

- Know about special students and various techniques which will meet their special educational needs.
- Comprehend the steps in TIGER method.
- Acquire a clear perspective of Active learning methodology.
- Understand Analytic and synthetic methods with illustrations.
- Identify the merits and demerits of Analytic and synthetic methods.
- Compare Analytic and synthetic methods of teaching mathematics.

5.3 PROVISIONS FOR HETEROGENEOUS CLASSROOMS

Implementing an appropriate instruction to heterogeneous classroom becomes a challenge for a teacher to deliver a lesson content and context to his students. The expected instruction is to reduce the diversity and to create the equality among heterogeneous students in a classroom. With equality among them, they may work and learn together in their classroom. The teacher frequently finds the difficulties in teaching and learning because of heterogeneous classroom. Heterogeneous classroom is a classroom with heterogeneous students from different aspects such as socioeconomic backgrounds, ethnic backgrounds, and learning speeds.

Because of the heterogeneous classroom, the instructional ways become a crucial issue in the context of teaching and learning process in all subjects. This issue invites teachers to solve deficits in flexible instruction in a heterogeneous classroom than they only focus on a certain instruction.

How to design a classroom powerfully is dependent on how a teacher views teaching and learning process. Because teachers should view teaching and learning process in the classroom as an active of engagement process, redesigning a classroom is important to do. This attempt relates to heterogeneous students in the classroom. Because of it, teachers need to reexamine the desks, tables, and other facilities to improve the instructional ways in the classroom. Furthermore, a classroom actuates many varieties of activities, group study, individual study, one-on one discussions (interaction between a teacher and a learner), a classroom may be handled by a teacher and more than one teacher (team-teaching) and there is much to think about when arranging the classroom environment, arranging that includes not only the room, but also other contexts such as the media center, computer laboratory, and even places where field trips take place, so classroom management

refers to the teacher's ability to direct, organize, and facilitate the learning environment and student behaviour within a learning context.

Classroom is an important environment to learn. Learning environment is important to be activated with supporting interesting methods, well-preparations for exercises, presentations, thesis and essays. Learning environment in safe and trustful situation is a must to train, to study, and to practice students' skills.

Classrooms still become a challenge for a teacher if the classroom contains heterogeneous students. The heterogeneous classroom filled with students from different socioeconomic and ethnic backgrounds, and learning levels. Before designing the course in heterogeneous classroom, some information about the students which are necessary to be prior understood are the background of the students, students' proficiency level, and intercultural competence level of students, students' interest, students' learning preferences, and students' attitudes. Furthermore, in heterogeneous classrooms, teachers need to identify status problems. Without hesitation, teachers can name students who are vocal and who dominate, and others who seem withdrawn and rarely participate. Common explanations for these behaviours define them as manifestations of students' personalities: some are natural leaders, others are shy, teachers claim.

5.4 SPECIAL STUDENTS AND TEACHING OF MATHEMATICS

"There are many factors that affect a student's understanding of mathematics, from lack of confidence and poor memory, to being moved on to new work before sufficiently understanding the previous underlying concepts. For some students there may also be physical or sensory difficulties; for others there may just simply be gaps in their knowledge due to a change of school or missing lessons through illness.

Sometimes the issues may be similar to those of struggling readers, which can include specific learning difficulties or more general problems with concentration. Whatever the cause, there is always a solution."

In many instances, the perceived solution to a student's difficulty is to provide him with more opportunities to rehearse, revise or revisit basic mathematical concepts. More fundamental to the solution, is the teacher's identification, knowledge and understanding of the student's particular difficulty. Equipped with this information, the teacher will be better positioned to develop an appropriate intervention programme for the student, to advise the parents on how they can support their student's learning and to explain to the student the nature of his difficulty and the strategies he can use to take control of that difficulty.

There are some practical suggestions, strategies and general good practice in relation to mathematics teaching. In applying this advice in your work with a student who is experiencing difficulty thinking with numbers, it will be important to select those aspects which best suit the individual student's needs, ensuring that the planned intervention programme allows for an appropriate balance between practical and written work which is underpinned by the student's understanding of the mathematical concept.

Inherent in the programme should be the opportunity for the student to verbalise what he is thinking and doing. The teacher has a crucial role to play in explicitly modeling thinking, desired strategies and the use of accurate mathematical language.

Everyone knows the feeling of struggling with a task that other people seem to understand thoroughly. This, of course, is how some students feel about maths and their difficulties are often rooted in misunderstanding of concepts. Understanding our mistakes can be powerful learning experience.

Dyscalculia is the inability to understand mathematical concepts and this impacts across the whole of a student's mathematical learning. There are many forms of dyscalculia and the two types most commonly found in school are:

DEVELOPMENTAL DYSCALCULIA – where a student's potential is not met by their attainment in mathematics

DYSCALCULIA – inability to manage mathematical concepts across a complex wide range of areas for example, inconsistent results in addition, subtraction, multiplication and division alongside poor mental mathematics ability. Common mistakes often include difficulties with writing, reading and recalling numbers as well as number additions, substitutions, transpositions, omissions, and reversals. There may also be an inability to grasp and remember mathematical concepts, rules, formulas and sequences. Poor long term memory may also be an issue i.e. they may be able to perform mathematical operations one day, but not on a subsequent day. They may lack the "big picture / whole picture" thinking.

It is our ambition that every student with Special Educational Needs reaches their full potential in school, and can make a successful transition to adulthood and the world of further and higher education, training or work. To promote the welfare and interests of such students and to improve the support they receive, there now follows practical ideas and advice on how, we as teachers, can raise the levels of achievement and ignite a mathematical imagination for such pupils in our care.

NOTES

GOOD PRACTICE

1. Maintaining Teacher-Parent- Students Relationship

Living with, or teaching a student who has difficulty thinking with numbers can be an emotionally charged experience. Frustration and confusion can complicate the conversation between parents and teachers about what to do. Respect for each other and open communication can reduce tension and enable parents and teachers to benefit from each other's expertise and knowledge of the student from different perspectives. Working together, parents, teachers, and the students themselves, can inform one another about how best to address the student's needs.

- Share observations of the student's mathematics profile and discuss where the breakdown is occurring.
- Identify and discuss the student's strengths and interests.
- Explain the student's difficulties to the parents.
- Clarify the intervention program.
- Advise parents on how to support their student's learning.

2. Talk with the student about their strengths and weaknesses

These students often give up and see themselves as failures while others exhibit behaviour complications. The following suggestions can help parents and teachers work together to demystify students's difficulties with mathematics.

- Discuss strengths and interests.
- Articulate clearly for the student the exact nature of his/her difficulty.
- Provide the student with strategies to manage the difficulty.
- Emphasise optimism.
- Identify a Maths Mentor.
- Eliminate any stigma.
- Protect from humiliation.

3. Classroom Practice (Suggestions and Strategies)

- Every day is mental maths day- introduce each session with a few minutes mental activity recording on white boards.
- Identify students with SEN in mathematics early on through appropriate diagnostic assessment and ensure that they receive early intervention.
- Allow time for talking about mathematics to clarify and refine thinking.

- Make learning as active and fun as possible – a positive experience.
- If there are no co-existing reading difficulties, encourage the pupil to read problems aloud.
- Ensure students can participate with confidence.
- Build on the pupil's existing knowledge.
- Understand the pupil's mistakes – looking in depth at the errors.
- Learning from mistakes should build up students's confidence.
- Provide regular and appropriate practical activities to help the student develop an understanding of number bonds so that automaticity is achieved.
- Over- learn basic techniques.
- Limit copying from the board.
- Provide flow diagrams or tree diagrams for clarifying procedures.
- Teach basic concepts using concrete objects.
- Allow them to 'wean' themselves off concrete methods as their confidence and understanding increases.
- Provide specialized materials for example squared paper, highlighters, Cuisenaire rods, base – ten blocks, number – lines, multiplication tables, etc.
- Make your expectations explicit along with success criteria.
- Provide time for checking work.
- Give students opportunities to connect mathematical concepts to real- life situations.
- Lots of practical repetition.

Improving Memory

- Provide the technology (ICT) and tools needed for problem-solving.
- Teach basic mathematics facts.
- Use a personal mathematics rule book where the pupil has, in their own words, recorded strategies and mathematics vocabulary.
- Teach mathematics in a variety of learning styles.
- Use games to reinforce concepts.
- Practice little and often

NOTES

Simplifying Language

- Teach mnemonic strategies for solving word problems.
- Focus on the information provided in word problems.
- Encourage students to put problems into their own words.
- Teach and constantly model the use of accurate mathematical language.
- Encourage pupils to teach a concept to aid understanding.

Enhancing Attention

- Students create a reminder card to keep on their desk or in their mathematics work book for quick reference to the strategy.
- Teach students how to self – monitor.
- Allow time for 'Brain Breaks'
- Model how to carry out a task.
- Teach self – checking strategies (eg: use of calculator).
- Identify topics of interest to students.
- Isolate steps – have the students focus on one step at a time.
- Ensure completion of each step in sequence.
- Ensure work sheets are uncluttered to ensure that the page does not look intimating (if required, cover unnecessary text / diagrams etc.).

Encouraging Self-organisation

- Teach students how to plan a task.
- TIPS: Think (read and paraphrase), Information (what numbers and information do you need in order to solve the problem?), Problem (write equation), Solve.
- Stress the importance of organization – demonstrate, rehearse and review.
- Encourage self – evaluation.
- Set goals and record progress.
- Practice estimating.
- State the amount of time a task should take to slow down students down or to speed them up.
- Provide consistent, specific, positive and constructive feedback.

NOTES

Check Your Progress

Notes : a) Write your answers in the space given below

b) Compare your answer with the one given at the end of the unit

1. What is Heterogeneous classroom?

   ……

   ……

2. What do you mean by Dyscalculia?

   ……

   ……

5.5 ACTIVE LEARNING METHODOLOGY

Today's students are tomorrow's heroes. So teachers and schools are taking important role in moulding the future buds of our nation. So in order to create interest in learning, the method of teaching is very important. Active learning Methodology is one of the teaching method. Active learning is involving students directly and actively in the learning process itself.

Active Learning method [ALM]

Active learning methodologies require that the student must find opportunities to meaningful talk and listen, write read and reflect on the content, ideas, issues and concerns of an academic subject.

Bonwell and Eison [1991] state that some merits of active learning are;

- ❖ Students are involve in more than listening.
- ❖ Less emphasis is placed on transmitting information and
- ❖ Greater emphasis on developing student's skills
- ❖ Students are involved in higher order thinking [analysis, synthesis, and evaluation]
- ❖ Students are engaged in activities [e.g. reading, discussing, and writing]
- ❖ Greater emphasis is placed on student's exploration of their own attitudes and values.

Active learning shifts the focus from the teacher to the student and from delivery of subject content by teacher to active engagement with the material by the student. Through appropriate inputs form the teacher, students learn and practice how to apprehend knowledge and use them meaningfully.

- ❖ The educator strives to create a learning environment in which the student can learn to restructure the new information and their prior knowledge into new knowledge into new knowledge about the content and to practice using it.
- ❖ Students are assumed to be an intelligent participant in knowledge creation who can lookup definition before and after class independently.
- ❖ Students can develop skills in constructing and using knowledge with the educator's guidance, alone and also with others in small and large groups.
- ❖ The educator may explain concepts, principles and methods.
- ❖ Visual aids, demonstrations, etc., integrated into class presentations.
- ❖ Students have the opportunity to remember upto 50% of the content of each class session.
- ❖ Students care deeply about their own education.
- ❖ Students learn to monitor and discuss their own learning.
- ❖ Students collaborate with other students to discover and construct a framework of knowledge that can be applied to new situations.

Scope of ALM

The aim of ALM is empowerment of the learner in such a way that he or she is confident and able to function in many contexts. In the middle school years [classes 6-8], such learning can be blended into the curriculum of any school easily. It includes:

- ❖ Learning to affirm oneself and one's learning style.
- ❖ Learning to be healthy and safe – Biology curriculum enrichment.
- ❖ Learning to think skillfully, recognize and deal with one's feelings and be resourceful in a variety of situations. Units of Learning for Life.
- ❖ Leaning to live in social system-living and working together with other people, good citizenship skills and being able to participate in debate and discussion.
- ❖ Learning to live in and interact with a physical environment-finding environmentally viable responses in terms of lifestyle and choices.
- ❖ Developing leadership and personality among all students in classes 6 to8.

Above all, Active Learning Skills will help students negotiate the world of knowledge with competence and enthusiasm, confident of their own abilities and opening widening newer avenues to learning.

Enrichment Activities

- ❖ Supply of DVDs depicting ALM model classes
- ❖ Training and supply of Audio and Video CDs to promote the basic skills
- ❖ Supply of modules to facilitate teachers for interesting and challenging homework, which extend the classroom processes to home.
- ❖ Conducting workshops to promote hands-on learning experiences and experiments.

Advantage of ALM

- ❖ Active engagement on the student's par.
- ❖ Provides a template for learning and learning to learn
- ❖ The student is not subjected to endless passivity
- ❖ Applicable in large classrooms and schools with few teachers
- ❖ Requires no special aids or special equipment
- ❖ Students can be resources for each other through paired and group activity
- ❖ The teacher can devote some time to students who need special help
- ❖ Allow the student to check her/his work against the teacher and thus save the teacher endless corrections while ensuring accuracy in Students learning.
- ❖ Works at student friendly and realistic assessment formats

The beauty of the process is its simplicity. Allows room for all student's voices to be heard through discussions and presentations.

5.6 TIGER METHODOLOGY IN MATHEMATICS TEACHING

In this modern age learning plays an important role. Mathematics has derives role in building up our civilization. It is more important for the common man also. Mathematics has not only been useful; in its own right but it has also enriched this world by helping in development of other fields of knowledge. So it is important to learn mathematics and develop the skills. Active learning methodology make the students as intelligent participants in knowledge creation. The format used for

mathematics teaching in Active learning Methodology is TIGER FORMAT. It is skill oriented.

Aims of TIGER Method

How Mathematics learning should occurinorder to capture the mind of every pupil? How it should reinforce their learning? are the two aims which is focused today. So to achieve these aims it is necessary to introduce the TIGER FORMAT of Active Learning Methodology.

The following are some of the methods adopted in Active Learning Methodology

- Self-study, Group study, Co-operative learning with teacher and Programmed learning
- Learning by understanding
- Meaningful learning
- Self-learning by the students
- Self designing of learning by the students.

Time duration for Active Learning Methodology is 90 minutes. Of this 90 minutes teacher should think of how to solve the problem? Why the solution should be found? The 'TIGER FORMAT' gives a clear picture of how to use each and every seconds of the class effectively.

Importance of TIGER Method in Mathematics

Normally teachers explain the problems by writing it on the black board. Students copy it in their note book. Some problems related to what was taught isgiven as Home Work. In the class room students do not get any opportunity to analyse and concentrate more on the problems. When they are doing it as home work, they find it difficult to overcome their doubts if any. There is also no opportunity to discuss with the peer group. So mathematics will be bitter for them as they do not understand it.

So inorder to overcome this bitterness Active Learning Methodology has been introduced in Mathematics. Here the students get opportunities to discuss with the peer group as well as the concerned teacher.

We Indians introduced '0' but after India produced a very few Mathematicians like Ramanujan and SivasankaranPillai. We can also introduce new great mathematicians in our schools too by using Active Learning Methodology in our class rooms.

TIGER FORMAT

- Expansion of this format is as follows:
- T-Teacher as a facilitator

- I-Individual work
- G-Group work
- E-Evaluation
- R-Reinforcement

Main features of TIGER

When the students are learning Mathematics through TIGER FORMAT

- They realize the happiness of learning mathematics.
- They realize that mathematics is not only with formulas and theorems.
- They could speak about mathematics as well they could speak through mathematics.
- They could solve even the tough problems by their own.
- They realize the relationship among each topics.
- They learn to generalize the knowledge of mathematics only by learning TIGHER FORMAT

Steps in TIGER FORMAT

It has some steps to follow.

1. Introduction

In this step the teacher should introduce the topic under three heading.

Evocation:

The teacher should evocate the students by giving puzzles, mathematical games, life history of mathematicians, interesting incidents in their life etc. Care should be taken whether it is related to the topic.

Recall :

The teacher ask some questions to check the previous knowledge of the students. The teacher develops the thinking capability of the students which leads to the topic of the day.

Survey:

The teacher directs the students to check the page number and the topic to be taken. The students have a glance on the main headings, sub headings, diagrams and definitions. The teacher should confirm that all the students have the text book with them.

2. Understanding

It gives importance to the knowledge of concept and the development of problems solving skills.

NOTES

Concept

The teacher should clearly explain the definitions and formulas needed to the class and write it on the black board.

Teacher Solving Problem

The teacher direct the students to analyse what is given? What is to be finding out in the problem? By giving step by step explanation, the teacher should develop the following skills among the students:

- Conceptual skill
- Sequential skill
- Arithmetic skill

Individual Solving Problem

The teacher ask the students to do the similar problem in the exercise done by him individually.

3. Group Work

Group work is an important one for mathematics learning. Discussions always help us to proper understanding of the problem.

Challenging Problems

The teacher gives the challenging problems to the students in group wise.

A group contains five to six students only. The problem should develop the thinking ability of the students. The teacher clear their doubts by orally or writing on the black board.

Group Activity

The group activity given should be related to the topic. The teacher supervises the groups whether they are doing the work correctly. After finishing, the small groups present their activity to the large group.

Presentation

An average student from each group presents their problem on the black board. After finishing the problem, all students are asked write it on their note book.

5. Evaluation :

Evaluation is an important task in the teaching-learning process.

Large Group Reinforcement

The teacher writes the formula and concepts on the black board. The teacher makes the pupil to draw mind map and flow chart.

Home work

The simple problems in the exercise are given as home work. If needs the teacher gives hints also.

Check Your Progress

Notes : a) Write your answers in the space given below

b) Compare your answer with the one given at the end of the unit

3. Write a short note on Active Learning method [ALM]

……

4. What is Expansion of TIGER format?

……

……

5.7 ANALYTIC METHOD

The word 'analytic' is derived from the word 'analysis', which means 'breaking up' or resolving a thing into its constituent elements. This method is based on analysis and, therefore, in this method we break up the problem in hand into its constituent parts so that it ultimately gets connected with something obvious, or already known.

In analytic method we proceed from unknown to known, from abstract to concrete and from complex to simple.

EXAMPLE – 1

If $\frac{a}{b}$ $\frac{c}{d}$ prove that $\frac{ac + 3b^2}{bc} = \frac{c^2 + 3bd}{dc}$

To prove this using analytic method, begin from the unknown.

The unknown is $\frac{ac + 3b2}{bc} = \frac{c2 + 3bd}{dc}$

$\frac{ac + 3b^2}{bc} = \frac{c^2 + 3bd}{dc}$ is true

if $\frac{ac + 3b^2}{b} = \frac{c^2 + 3bd}{d}$ is true

if $d(ac + 3b^2) = b(c^2 + 3bd)$

if $dac + 3b^2 d = bc^2 + 3b^2 d$

if $dac = bc^2$

if $\frac{a}{b} = \frac{c}{d}$ which is given to be true

$$/ \quad \frac{ac + 3b^2}{bc} = \frac{c^2 + 3bd}{dc}$$

EXAMPLE : 2

Prove that the diagonals of a parallelogram bisect each other.

What is to be proved will be placed before the students. The relevant diagram will be drawn and then analysis will proceed in the way given ahead:

1. What is given?

   ABCD is a parallelogram. AC and BD are its two diagonals which intersect each other at O.

2. What is to be proved?

   AC and BD bisect each other.

3. What parts in the figure, therefore, are to be proved equal?

   AO = OC and BO = OD

4. How can you prove these parts equal?

   If we can prove ΔAOD and ΔBOC or ΔAOB and ΔCOD as congruent.

5. How can you prove ΔAOB and ΔCOD as congruent?

   For this we will have to think about the conditions of proving two triangles congruent.

6. In view of these conditions what elements, therefore, should be proved equal?

   We have to prove two angles and a side of both the triangles as equal.

7. In what different situations may two angles be proved equal?

   They should be vertically opposite angles, right angles, alternate or corresponding angles.

8. What pairs of the angles are possible in the present situation?

∠AOB and ∠COD are vertically opposite angles and other pair may be of alternate or corresponding angels.

9. How do alternate or corresponding angles appear in this figure?

   When AB and CD are parallel.

10. When may AB and CD become parallel?

    When ABCD is a parallelogram.

In this way by starting from what is to be proved one comes to what is given. Then analysis helps in discovering what is needed in proving the given statement.

MERITS OF ANALYTIC METHOD:

- It leaves no doubts in the minds of the students as every step is justified.
- It is the psychological method.
- It facilitates clear understanding of the subject matter as every step is derived by the student himself.
- It helps in developing the spirit of enquiry and discovery among the students.
- No cramming is necessitated in this method as each step has its reason and justification.
- Students take active role in the learning process resulting in longer retention and easier recall of what they learn.
- It develops self-confidence in the students as they tackle the problems confidently and intelligently.
- It develops thinking and reasoning power among the students.

DEMERITS OF ANALYTIC METHOD

- It is a lengthy, time consuming method and therefore not economical.
- With this method it is difficult to acquire efficiency and speed.
- This method may not be suitable for all topics of mathematics.
- In this method information is not presented in a well organized manner.
- This method may not be very effective for below average students who would find it difficult to follow the analytical reasoning.

SYNTHETIC METHOD

'Synthetic' is derived from the word 'Synthesis'. Synthesis is the complement of analysis. To synthesis is to combine the constituent elements to produce something new. In this method we start with something already known and connect it with the unknown part of the statement.

In synthetic method one proceeds from known to unknown. It is the process of combining known bits of information to reach the point where unknown information becomes obvious and true.

EXAMPLE – 1

If $\frac{a}{b}$ $\frac{c}{d}$ prove that $\frac{ac + 3b^2}{bc} = \frac{c^2 + 3bd}{dc}$

EXAMPLE : 2

Prove that the diagonals of a parallelogram bisect each other.

Given: ABCD is a parallelogram. AC and BD are two diagonals which intersect each other at O.

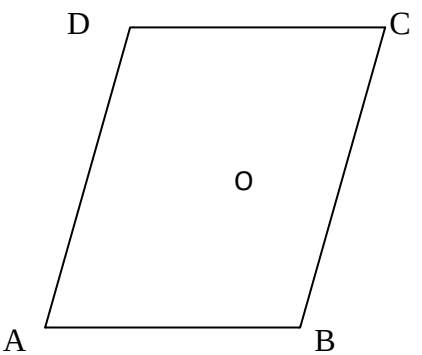
To prove: AO = OC and BO = OD

Proof: In ΔAOB and ΔCOD

∠AOB = ∠COD (Vertically opposite angles)

∠ABO = ∠ CDO (Alternate angles)

And AB = CD (Opposite sides of a parallelogram)



ΔAOB and ΔCOD are congruent.

AO = OC and BO = OD

MERITS OF SYNTHETIC METHOD:

- This methods is logical as in this method one proceeds from the known to unknown

- It is short and elegant
- It facilitates speed and efficiency
- It is more effective for slow learners.

DEMERITS OF SYNTHETIC METHOD:

- It leaves many doubts in the minds of the learner and offers no explanation for them.
- As it does not justify all the steps, recall of all the steps may not be possible.
- There is no scope for discovery and enquiry in this method.
- It does not provide full understanding.
- It makes the students passive listeners and encourages rote memorization.
- If the student forgets the sequence of steps, it would be very difficult to reconstruct the proof/solution.

METHODS COMPARISON OF ANALYTIC AND SYNTHETIC

Analytic Method	Synthesis Method
▪ Analysis means breaking up into components	Synthesis means combining the elements to get something new
▪ Leads from unknown to known.	Leads from known to unknown
▪ A method of discovery and thought	A method for the presentation of discovered facts.
▪ Lengthy, laborious and time consuming	Short, concise and elegant.
▪ Valid reasons to justify every step in the sequence	No justification for every step in the sequence.
▪ Encourages meaningful learning	Encourages rote learning
▪ Easy to rediscover	Once forgotten, not easy to recall
▪ Encourages originality of thinking and reasoning	Encourages memory work
▪ Is formational	Is informational
▪ A psychological method	A logical method
▪ Application of inductive reasoning	Application of deductive reasoning
▪ Informal and disorganized	Formal, systematic and orderly

▪ Process of thinking	Product of thinking
▪ Active participation of the learner	Learner is a passive listener

Check Your Progress

Notes : a) Write your answers in the space given below

b) Compare your answer with the one given at the end of the unit

5. What is the meaning of analytic method?

……

……

6. Mention any two demerits of synthetic method.

……

……

5.8 LET US SUM UP

Classrooms still become a challenge for a teacher if the classroom contains heterogeneous students. The heterogeneous classroom filled with students from different socioeconomic and ethnic backgrounds, and learning levels. Before designing the course in heterogeneous classroom, some information about the students which are necessary to be prior understood are the background of the students, students' proficiency level, and intercultural competence level of students, students' interest, students' learning preferences, and students' attitudes.

Active learning shifts the focus from the teacher to the student and from delivery of subject content by teacher to active engagement with the material by the student. Through appropriate inputs form the teacher, students learn and practice how to apprehend knowledge and use them meaningfully.

Analytic and synthetic methods are two such methods which in spite of their opposing nature are used in combination. Analysis must take lead and necessarily be followed by synthesis for realizing the better results in teaching of mathematics.

5.9 UNIT-END ACTIVITIES

1) Discuss briefly the Active Learning Methodology.
2) What do you mean by special students and how could you teach mathematics for them?
3) Explain TIGER methodology in mathematics teaching.

4) Illustrate and discuss Analytic and synthetic methodsof teaching mathematics.

5) Write the advantages and disadvantages of synthetic method.

6) Compare Analytic and synthetic methodsof teaching mathematics.

5.9 ANSWERS TO CHECK YOUR PROGRESS

1. Heterogeneous classroom is a classroom with heterogeneous students from different aspects such as socioeconomic backgrounds, ethnic backgrounds, and learning speeds.

2 Dyscalculia is the inability to understand mathematical concepts and this impacts across the whole of a student's mathematical learning.

3 Active learning methodologies require that the student must find opportunities to meaningful talk and listen, write read and reflect on the content, ideas, issues and concerns of an academic subject. Active learning shifts the focus from the teacher to the student and from delivery of subject content by teacher to active engagement with the material by the student. Through appropriate inputs form the teacher, students learn and practice how to apprehend knowledge and use them meaningfully.

4 Expansion of TIGER format is as follows:

- T-Teacher as a facilitator
- I-Individual work
- G-Group work
- E-Evaluation
- R-Reinforcement

5 The word 'analytic' is derived from the word 'analysis', which means 'breaking up' or resolving a thing into its constituent elements. This method is based on analysis and, therefore, in this method we break up the problem in hand into its constituent parts so that it ultimately gets connected with something obvious, or already known. In analytic method we proceed from unknown to known, from abstract to concrete and from complex to simple.

6 Demerits of Synthetic Method:

- It leaves many doubts in the minds of the learner and offers no explanation for them.
- As it does not justify all the steps, recall of all the steps may not be possible.

5.11 SUGGESTED READINGS

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UNIT – VI
TEACHING FOR PROBLEM SOLVING

Structure

6.1 INTRODUCTION

Problem solving method as a method of teaching represents a method which provides opportunity to the student for analysis and solving a problem faced by him on the basis of the previous stock of his knowledge enriched with the present means available to him, quite independently by following some systematic and scientific steps and arriving at some basic conclusions or results to be utilized in future for the solution of the similar problems in the identical situations.

This unit brings out the mechanism, use of this method in mathematics, and polya's four steps in problem solving. The present unit presents the inductive and deductive methods of teaching mathematics which will help the teacher to plan instruction in the classroom in the most effective manner.

The National Policy of Education (1986) lays down the importance of mathematics as a vehicle for developing creativity. This unit discusses about creative learning and teaching and its components.

6.2 OBJECTIVES

At the end of the unit, the student-teachers will be able to

- Define problem.
- Enumerate polya's steps in problem solving.
- Understand the steps involved in inductive method.
- Understand the steps involved in deductive method.
- Compare inductive and deductive method.
- Know how to teach mathematics creatively.

6.3 PROBLEM-SOLVING METHOD

Problem solving method, as the name indicates, begins with the statement of a problem that challenges the students to find a solution. Problem solving method is popular method of teaching mathematics. Their problem itself is crux of problems. So, what is problem?

A problem is an obstruction of some sort to the attainment of an objective, a sort of difficulty which does not enable the individual to reach a goal easily. The problem centres around the subject matter under study and require the use of information and skills available to the students. In the process of solving the problem the students may be required to gather data, analyse and interpret the information, to arrive at a solution to the problems.

According to Yoakman R. Simpson, "Problem occurs in a situation in which a felt-difficulty to act is realized. It is a difficulty that is clearly present and recognized by the thinker. It may be purely mental difficulty or it may be physical and involve manipulation of data. The distinguishing thing about a problem however is that it imposes the individual who needs it as needing a solution. He recognizes it as a challenge".

6.3.1 Characteristics of Problem

Understood by above explanation, is as follows:

(1) It is a felt-difficulty.

(2) It may be mental or physical.

(3) Individuals take it is a challenge.

Life is arena of problems. At every stage of life, every individual, be it small child or adolescent or old persons, has to face problems of one form or other.

"The only worthwhile life is a life which contains its problems to live without any longings and ambitions is to live only half-way." is well said by L.A. Averill Problems can come from society – corruption,

population, degradation of values, unemployment, joint families, breaking families, education problem, marriage problem….

One is successful, when one faces all those problems with courage and patience.

Individual is in school for nearly 6-7 hours. Teacher is second mother of child. So,teacher can and should train children in art and craft of solving and facing problems of his life.

Problem solving method aims at presenting the knowledge to be learnt in form of a problem. Problems are given to students, during teaching/learning process in a natural way.

Mathematics is a subject which has variety of problems in it. Mathematics is also a subject, which has direct relation to practical life. When students study mathematics, keeping positive attitude, they become efficient and able enough to solve the problems.

6.3.2 Essential Qualities of a Problem

(1) Problems should be related to social environment of the students, so that they feel it easy to start with.

(2) It should be in line with needs and interest of a particular group of students.

(3) Problems should be related to daily life of students.

(4) Problems should be valuable.

(5) Language of problems should be simple, practical and clear.

(6) What is to be found out should be clearly evident in given problem.

(7) It should impart functional and rich learning.

(8) Problem should be in accordance to mental capacity of the learner.

(9) Students should be motivated to think and evaluate problem themselves.

(10) If problem is it can be solved in part.

(11) Teachers should use different medias like coloured chalks cut-outs, charts, slides, symbolic chart, tabular chart to motivate students.

(12) Teachers should be expert in the field.

(13) Teachers should be of helping and motivating nature.

(14) Students should conclude and generalize problems specifically.

These conclusions and generalizations, should be stated properly, so that these can be further used in the solutions of new problems.

6.3.3 Definition of Problem Solving

Risk, T.M defines problem solving as "Planned attack upon a difficulty or perplexity for the purpose of finding a satisfactory solution". Risk further elaborates that problem-solving teaching procedure is a process of raising a problem in the minds of students in such a way as to stimulate purposeful, reflective thinking in arriving at a rational solution.

According to James Ross "Problem solving is an educational device whereby the teacher and the students attempt in a conscious, planned, purposeful manner to arrive at an explanation or solution to some educationally significant difficulty."

Therefore, as used in teaching-learning situation, problem-solving is a method in which the felt difficulty to act in an educational situation is realized and then an attempt is made in a conscious and purposeful way to find its solution.

6.4 Polya's Problem Solving Techniques

George Polya described the experience of problem solving in his book, How to Solve It. As part of his work on problem solving, Polya developed a four step problem solving process. They are: 1. Understanding the Problem, 2. Devising a Plan, 3. Carrying out the Plan and 4. Looking Back.

1. Understand the Problem

- First. You have to understand the problem.
- What is the unknown? What are the data? What is the condition?
- Is it possible to satisfy the condition? Is the condition sufficient to deter- mine the unknown? Or is it insufficient? Or redundant? Or contradictory?
- Draw a figure. Introduce suitable notation.
- Separate the various parts of the condition. Can you write them down?

2. Devising a Plan

- Second. Find the connection between the data and the unknown. You may be obliged to consider auxiliary problems if an immediate connection cannot be found. You should obtain eventually a plan of the solution.
- Have you seen it before? Or have you seen the same problem in a slightly different form?

- Do you know a related problem? Do you know a theorem that could be useful?
- Look at the unknown! Try to think of a familiar problem having the same or a similar unknown.
- Here is a problem related to yours and solved before. Could you use it? Could you use its result? Could you use its method? Should you introduce some auxiliary element in order to make its use possible?
- Could you restate the problem? Could you restate it still differently? Go back to definitions.
- If you cannot solve the proposed problem, try to solve first some related problem. Could you imagine a more accessible related problem? A more general problem? A more special problem? An analogous problem? Could you solve a part of the problem? Keep only a part of the condition, drop the other part; how far is the unknown then determined, how can it vary? Could you derive something useful from the data? Could you think of other data appropriate to determine the unknown? Could you change the unknown or data, or both if necessary, so that the new unknown and the new data are nearer to each other?
- Did you use all the data? Did you use the whole condition? Have you taken into account all essential notions involved in the problem?

3. Carrying out the Plan

- Third. Carry out your plan.
- Carrying out your plan of the solution, check each step. Can you see clearly that the step is correct? Can you prove that it is correct?

4. Looking Back

- Fourth. Examine the solution obtained.
- Can you check the result? Can you check the argument?
- Can you derive the solution differently? Can you see it at a glance?
- Can you use the result, or the method, for some other problem?

Check Your Progress

Notes : a) Write your answers in the space given below

b) Compare your answer with the one given at the end of the unit

1. What is a problem?

   ……

   ……

2. State five essential qualities of a Problem

   ……

   ……

3. Define problem solving

   ……

   ……

4. What are the steps involved in Polya's problem solving?

   ……

   ……

Merits of Problem Solving Method

The following are the merits of problem solving method:

- Problem solving provides a real life like experience to the children.
- It develops in students good habits of planning, thinking, reasoning and independent work.
- It develops initiative and self-responsibility among the students.
- It takes into account individual differences.
- It helps the students to develop reflective thinking.
- It helps the students to approach future problems with confidence.
- It builds a mental attitude for effective learning based on critical thinking.
- It helps the children developmental traits of open-mindedness and tolerance as the children see many sides to a problem and listen to many points of view.

Demerits of Problem Solving Method

The following are the demerits of Problem solving method:

- Not all students are problem solvers.
- The problem solving method becomes monotonous if used too frequently.

- It is time consuming and consequently it is not possible to cover the syllabus on time.
- The success of this method depends upon mathematics teachers who are well versed in critical and reflective thinking. Not all mathematics teachers are well versed in those types of thinking.
- Reference and Resource materials may be difficult to come by.
- Only a skilled and resourceful teacher will be able to make an effective use of this method.
- All topics in mathematics cannot be taught through this method.
- Textbooks are not available according to this method.
- Lack of interest and motivation on the part of the students can spoil the effectiveness of this method.

Teachers' Role in Problem Solving Method

Teacher plays a significant role in problem solving method. The teacher's role is to:

- Ensure an atmosphere of freedom in the class.
- Create the problem situation.
- Help the students in accepting, defining and stating the problem.
- Help the students in analysing the problem and in breaking up the problem into simple units.
- Help the students keep their attention focussed on the main problem all the time.
- Assist the students in locating relevant source materials.
- Encourage the students in seeking important relationships in the data.
- Help the students develop an attitude of open mindedness and critical enquiry.
- Exhibit spirit of enquiry and discovery.

How to Teach Students to Solve Problems?

There are two devices: the illustrative examples and the set of directions. The first works out one problem in detail. This is followed by practice problems of the same type. This does not develop original thinking, because the student generally follows the steps mechanically.

Explaining a problem is a very different thing from teaching it. The illustrative problem has a place in the teaching procedure because the student may refer to it in case of need. The student should be urged to work them with the book closed.

The solving of verbal problems requires a variety of activities. The situations of the problem should be properly understood. Facts must be identified and expressed in symbols and much practice should be given in it. The relationships involved are to be clearly stated in words and expressed in symbols. Try to make a problem without the actual numbers and estimate the result in advance. Relationships that are not states must be supplied by the student. Formulae are to be recalled and evaluated. Numerical values are to be substituted in the formulae. Equations are to be derived and solved. The solutions are to be checked. All of this calls for exactness and for training in reading, planning, thinking, making decisions, and drawing inferences.

Check Your Progress

Notes : a) Write your answers in the space given below

b) Compare your answer with the one given at the end of the unit

5. What are the merits of problem solving method?

……

……

6. What is the teachers' role in problem solving method?

……

……

6.5 Inductive Method

Inductive Method is based on induction. Induction is the process of proving a universal truth or a theorem by showing that if it is true of any particular case, it is true of the next case in the same serial order and hence true for any such cases. So the technique of making transition from particular facts to generalisations about these facts is known as the process of induction. Thus it is a method of arriving at a formula or a rule by observing a sufficient number of particular instances. If one rule applies to a particular case and is equally applicable to different similar cases, it is accepted as a general rule or formula. Therefore, in this method we proceed from particular to general, from concrete instances to abstract rules and from simple examples to complex formula. A formula or a generalisation is arrived at through inductive reasoning.

This method has been found to be very suitable for teaching of mathematics because many mathematical formulae and generalisations are the results of induction.

Example -1

Draw a set of parallel lines and ask students to measure corresponding and alternate angles. They will conclude corresponding and alternate angles are equal.

Example -2

The sum of the angles in a triangle is 180^0.

When a student measures the angles of several triangles and finds that in each case the sum of angles approximates to 180^0, he has the background to generalise that sum of the interior angles in a triangle is 180^0.

Example - 3

The volume of a cone is one third of that of a cylinder

Volume of a cone = $\frac{1}{3}\pi r^2 h$. This can be derived by inductive method.

Take two containers one conical in shape and the other cylindrical in shape, having the same height and diameter. Fill the conical vessel with saw dust and empty it into the cylindrical vessel. Repeat it with similar vessels of varying dimensions. It can be seen that each time it takes three full cones to fill the cylinder. Thus leads to the conclusion that volume of the cone = $\frac{1}{3}$ of the volume of cylinder.

Volume of cone $=\frac{1}{3}\pi r^2 h$

Many more examples can be listed where inductive method can be used effectively. A few are given below.

- Laws of indices
- Properties of parallelogram, rhombus etc.
- Factorisation of quadratic expressions
- Derivation of the formula for simple interest, compound interest, recurring deposit etc.
- Transversal properties of parallel lines.
- Vertically opposite angles are equal.
- Angle in a semicircle etc.

Steps in Inductive Method

Inductive method follows clear and specific steps as under:

- Selection of a number of cases
- Observation of the case under given conditions.

- Investigation and analysis
- Finding common relations
- Arriving at generalisation
- Verification or application.

Merits of Inductive Method

- It helps understanding.
- It is a logical method and develops critical thinking.
- It encourages active participation of the students in learning.
- It provides ample opportunities for exploration and observation.
- It sustains the student's interest as they proceed from known to unknown.
- It curbs the tendency for rote learning as it clears the doubts of the students
- It facilitates meaningful learning.
- It enhances self-confidence.

Demerits of Inductive Method

- Its application is limited to very few topics in mathematics where actual observation of the particular instances is possible.
- This method is not suitable for higher classes because higher order mathematical principles cannot be generalised through the observation of concrete cases.
- It is lengthy, time-consuming and laborious method.
- It is not absolutely conclusive as it might leave some doubts in the minds of the students regarding the validity of the generalisation arrived at through the observation of a few particular instances.
- This method only facilitates the discovery of the formula or the rule. A lot of supplementary work and exercises is necessary to fix it in the mind of the learner.
- It is not suitable for mathematically gifted students as unnecessary details and too many examples make the teaching dull and boring.

Applicability of Inductive Method

Inductive method is most suitable where

- rules are to be formulated
- definitions are to be formulated
- formulae are to be derived.
- generalisations or laws are to be arrived at.

While selecting inductive method for teaching, a teacher should check whether it is possible to present sufficient number of particular cases as instances of the generalisation to be arrived at. The student should be in a position to draw the conclusions without doubting the chance occurrence of instances and thus doubting the validity of the generalisation itself.

6.6 Deductive Method

Deductive Method is based on deductive reasoning. Deductive reasoning is the process of drawing logical inferences from established facts or fundamental assumptions. In this method the teacher presents the known facts or generalisation and draws inferences regarding the unknown, following a network of reasoning. Therefore, in deductive method one proceeds from general to particular instances and from abstract to concrete cases. This approach is not suitable for exploration, but appropriate for a final statement of mathematical results.

In this method, we begin with the formula, or rule or generalisation and apply it to a particular case. It can illustrate by the following examples.

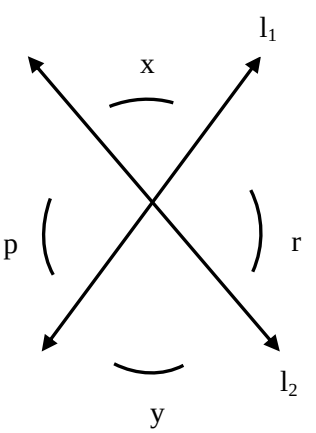
Example – 1

When two lines intersect, the vertically opposite angles are equal. (Statement of generalisation)

l_1 and l_2 are intersecting lines

x, y are vertically opposite angles.

Similarly p, r are vertically opposite angles.



The following inferences are drawn from the statement of the generalisation

x=y (Vertically opposite angles)

p = r (Vertically opposite angles)

Examples – 2

a^m x a^n = a^{m+n} (statement of the law)

Find x^2 x x^{10}.

By applying the law we have, x^2 x x^{10} = x^{2+10} = x^{12}

Whenever the teacher states a formula or a rule and asks the students to apply it to solve problems, the teacher is following the deductive method.

Steps in Deductive Method

Deductive method of teaching follows the steps given below.

- Clear recognition of the problem: A clear recognition of the problem statement provides the basic link for the thinking process and the solution to the problem.
- Search for a tentative hypothesis: The second step in deductive method is the search for a tentative hypothesis, a tentative solution to the problem.
- Formulation of a tentative hypothesis: The search for the solution leads to the formulation of a tentative hypothesis that appears to have promise as a possible or probable solution to the problem. The tentative hypothesis has its basis on certain axioms or postulates, or propositions or rules and formulae that have been accepted to be true.
- Verification: Finally the hypothesis that has been formulated is to be verified as the right solution to the problem at hand.

Merits of Deductive Method

- It saves time and labour for both the teacher and the student.
- It enhances speed, skill and efficiency in solving problems.
- It is a short and elegant method.
- It helps in fixation of formulae and rules as it provides adequate opportunities for practice and revision.
- It helps in increasing the memory power of the students, as the students are required to memorise a large number of laws, formulae etc.

- It completes the inductive method as probability in induction is reduced to certainty in deduction.
- It is useful for higher classes.

Demerits of Deductive Method

- It encourages rote memory as deductive method demands the use of certain laws, rules or formulae to be recalled by the learners from their memory.
- It does not clarify the doubts of the student regarding the generalisation and hence learning is incomplete.
- It is not suitable for beginners.
- It does not encourage students' involvement in learning.
- It is not suitable for the development of thinking, reasoning and discovery.
- Since it is based on blind memory, once the formula or rule is forgotten, it is not possible to rediscover them.
- In this method the emphasis is on memory at the cost of intelligence and hence it is psychologically unsound.
- It taxes the student's mind as it puts more emphasis on memory.

Applicability of Deductive Method

Deductive method is suitable for giving practice to the student in applying the formulae or principles or generalisation which has been already arrived at. This method is very useful for fixation and retention of facts and rules as it provides adequate drill and practice. Deductive method is particularly suitable for teaching demonstrative geometry as it is primarily a deductive science in which truths stated in the form of theorems can be proved by showing that they are implied by other theorems which have already been proved, definitions showing that they are implied by other theorems which have already been proved, definitions that have been stated and postulated and axioms that have been accepted. This method is best suited for teaching mathematics at higher classes. However, the deductive method is most effective when it is preceded by inductive method.

Comparison of Inductive and Deductive Method

Inductive Method	Deductive Method

• Based on inductive reasoning	• Based on deductive reasoning
• Proceeds from Particular -----> General Concrete -----> Abstract	• Proceeds from • General -----> Particular Abstract -----> Concrete
• A psychological method	• An unpsychological method.
• A method of discovery that stimulates intellectual powers	• A method of presentation that doesnot develop originality and creativity.
• Emphasis is on reasoning	• Emphasis is on memory.
• Encourages meaningful learning	• Encourages rote learning
• Most suitable for initial stages of learning	• Suitable for practice and application
• Suitable for lower classes	• Most suitable for higher classes.
• Enhances active participation of the students	• Makes the student passive recipient of knowledge
• Lengthy, time-consuming and laborious	• Short, concise and elegant
• Not absolutely conclusive, based on probability	• Makes the probability a certainty
• Facilities discovery of rules and generalisations	• Enhances speed, skill and efficiency in solving problems

6.7 Creative Teaching and Learning in Mathematics

Creativity in the mathematics classroom is not just about what students do but also what we do as teachers. If we are thinking creatively about the mathematical experiences we offer our students we can open up opportunities for them to be creative.

What is required of a mathematics teacher is "what is to be offer and how it is to be offered". In this regard, three things are of major concern.

- how we present content
- how we model good practice
- how we encourage our students to be creative

Mathematics is a creative subject and we as teachers need to be prepared to present it in more varied ways, including being prepared to

"let go" and give our students room to explore. In doing this we are not only allowing them access to what mathematics is really about -posing and solving problems - but also offering situations in which students can reveal their strengths and highlight areas where they need greater support.

What is Creative Teaching?

Teaching creatively means teaching with variations and innovations. A creative lesson is interesting, challenging, unconventional, productive, and motivating. There are variations in teaching techniques and materials, instructional activities, and assessment. There are innovations in designing teaching aids, selecting activities, and evaluation. Teachers can employ approaches that facilitate students' participation and active interaction. Creative mathematics teaching allows flexibility in adopting various pedagogical approaches that are suitable for students' level of understanding, and that are appropriate for the nature of the content.

Components of Creative Mathematics Teaching

- Creative teaching is influenced by various components. There are at least six components.
- The **first component** is related to basic pedagogical skills such as lesson planning, classroom management, communication, and evaluation.
- The **second component** refers to the domain specific expertise, creative techniques, and knowledge of developmental processes.
- The **third component** is related to the competence in selecting appropriate assessment modes.
- The **fourth component** refers to teachers' and students' motivation. Intrinsic motivation is an indispensable component as it generates an on-going commitment.
- The **fifth and the sixth components** are related to the learning climate and environment: Educational policies and the school culture. Creative mathematics teaching is influenced by educational policies and school cultures. Educational policies influence school learning climates.

Pedagogical Components

Creative teachers are able to co-ordinate various pedagogical components. Fundamental components are such as teachers' content knowledge, creative techniques, assessment modes, and individual needs.

Teachers' Content Knowledge

Teachers' understanding of mathematical concepts should be enriched with knowledge of child and adolescence psychology. Teachers should be aware of developmental processes of various age groups. They should investigate how students understand mathematical concepts. If teachers understand students' cognition and behaviours, they are likely to select appropriate teaching materials, and design suitable teaching aids.

For example inteaching geometry, teachers may find it useful to employ diagrams and objects. When learning processes involve hands-on experiences, students may understand new concepts easily. Cognitive development ofstudents is at the pre-operational and concrete operational stages (Piaget's model of cognitive development). Thus, learning with concrete objects can be helpful.

Creative Techniques

Teachers should acquire various creative techniques. Teachers should know conditions that stimulate creativity. The brainstorming technique, for instance, allows students to generate ideas in a non-threatening environment. Problem solving enables students to use information available to search for possible solutions. Creative techniques are modified according to students' needs and psychological development. In addition to describing in words, students should be allowed to use diagrams or symbols to represent their ideas. A teacher should infuse the brainstorming technique and the problem solving technique in teaching mathematics.

For example, in teaching isosceles triangle and equilateral triangle, the following learning experiences would enable creative learning. Studentscan be asked to discover possible features of an isosceles triangle and an equilateral triangle. First, studentscan be asked to use six straws with two equal lengths to form as many triangles as possible. Then they may be given colour cut-outs of two triangles (one is an isosceles triangle, and the other is an equilateral triangle). Studentsare asked to discover similarities and differences between them. They may be instructed to match triangles that they formed with the cut-outs, and to list as many similar features as possible. Studentsmay be allow to use diagrams to represent their answers.

Assessment Modes

Teachers should be experts in assessment. Formative and summative assessments are meant to find out students' learning progress at the end of a unit or a series of units. Diagnostic assessment can be used to find out weaknesses of individual students. Suitable contents and activities can then be designed. In addition to tests, teachers can employ other forms of assessment that allow independent and interdependent learning.

For example, a one-week project, designing a new school canteen may be given to students. In this project they would have the opportunity to apply their knowledge of area and perimeter. Students are guided to propose the most suitable location, built-in areas, arrangement of tables, chairs, and food stores.

Individual Needs

A creative mathematics lesson maintains students' learning interests, stimulates students' thinking, and encourages students to discover new knowledge. Individual needs of students are considered. It is indispensable to arouse students' interests in mathematics. Motivation is one of the prerequisite conditions for continuous learning and self-education. Mathematics can be an interesting subject, if teachers can associate this subject with students' pleasant learning experiences. Learning mathematics does not confine to memorising formulae. Games and quizzes that invite students' active participation can be infused into mathematical activities. Playing brings joy and fun. Quizzes, games, and puzzles with rewards are favourable activities of students. These activities provide them the opportunities to share, to discover, and to interact with peer. Learning that brings pleasant experiences is likely to generate satisfaction. When a lesson is interesting and enjoyable, it is likely that students develop positive attitudes towards learning mathematics.

Having information about students' level of understanding and needs, teachers are likely to define appropriate instructional objectives.The essential components of a creative mathematics lesson may be tabulated a follows.

Components of a Creative Mathematics Lesson

Planning and management	**Instructional activity**	**Creative expertise**
Lesson planning • Defining objectives • Preparing teaching aids • Determining teaching procedures Classroom Management • Behavioral management	Designing activity • Preparing activities – group of pair – that can stimulate various levels of understanding Conducting activity • Forming effective groups • Delivering effective	Mathematics • Fundamental knowledge • Possible misconceptions • Appropriate contents according to age groups Creative techniques • The inquiry approach

Planning and management	Instructional activity	Creative expertise
(e.g. movement, noise) • Time management Carrying a lesson • Explaining new concepts • Assigning homework • Giving feedback	instructions • Facilitating effective sharing Giving feedback • Facilitating immediate feedback • Encouraging self and peer evaluation • Integrating students' responses into new content	• Brainstorming • Problem solving Assessment • Identifying appropriate modes of assessment that capture students' strengths and weaknesses Identifying individual needs • Maintaining intrinsic interests • Stimulating thinking and discovery

Check Your Progress

Notes : a) Write your answers in the space given below

b) Compare your answer with the one given at the end of the unit

7. Write a short note on inductive method.

   ……

8. Mention any four merits of inductive method.

   ……

9. What are the steps involved in deductive method?

   ……

10. What are the components of creative mathematics teaching?

    ……

    ……

6.8 LET US SUM UP

There are four steps in Polya's Problem Solving Techniques. They are understand the problem, devising a plan, carrying out the plan and looking back.

Inductive Method is based on induction. Induction is the process of proving a universal truth or a theorem by showing that if it is true of any particular case, it is true of the next case in the same serial order and

hence true for any such cases. This method proceeds from particular to general, from concrete instances to abstract rules and from simple examples to complex formula. A formula or a generalisation is arrived at through inductive reasoning.

Deductive Method is based on deductive reasoning. Deductive reasoning is the process of drawing logical inferences from established facts or fundamental assumptions. In this method the teacher presents the known facts or generalisation and draws inferences regarding the unknown, following a network of reasoning. Therefore, in deductive method one proceeds from general to particular instances and from abstract to concrete cases.

Teaching creatively means teaching with variations and innovations. A creative lesson is interesting, challenging, unconventional, productive, and motivating. There are variations in teaching techniques and materials, instructional activities, and assessment. There are innovations in designing teaching aids, selecting activities, and evaluation. Teachers can employ approaches that facilitate students' participation and active interaction. Creative mathematics teaching allows flexibility in adopting various pedagogical approaches that are suitable for students' level of understanding, and that are appropriate for the nature of the content.

6.9 UNIT-END ACTIVITIES

1) What do you understand by problem method? What is its scope in the teaching of mathematics? How will you employ it?
2) Explain Polya’s problem solving techniques.
3) Which is the best method of teaching mathematics according to your option? Support your preference with arguments.
4) Illustrate and discuss the inductive-deductive methods of teaching mathematics.
5) As a teacher how do you develop problem solving skills among your students?

6.10 ANSWERS TO CHECK YOUR PROGRESS

1. A problem is an obstruction of some sort to the attainment of an objective, a sort of difficulty which does not enable the individual to reach a goal easily. The problem centresaround the subject matter under study and require the use of information and skills available to the students.
2. Essential Qualities in a Problem

   (1) Problems should be related to social environment of the students, so that they feel it easy to start with.

(2) It should be in line with needs and interest of a particular group of students.

(3) Problems should be related to daily life of students.

(4) Problems should be valuable.

(5) Language of problems should be simple, practical and clear.

3. Risk, T.M defines problem solving as "Planned attack upon a difficulty or perplexity for the purpose of finding a satisfactory solution". Risk further elaborates that problem-solving teaching procedure is a process of raising a problem in the minds of students in such a way as to stimulate purposeful, reflective thinking in arriving at a rational solution.

   According to James Ross "Problem solving is an educational device whereby the teacher and the students attempt in a conscious, planned, purposeful manner to arrive at an explanation or solution to some educationally significant difficulty."

4. Polya developed a four step problem solving process. They are: 1. Understanding the Problem, 2. Devising a Plan, 3. Carrying out the Plan and 4. Looking Back.

5. The following are the merits of problem solving method:

   - Problem solving provides a real life like experience to the children.
   - It develops in students good habits of planning, thinking, reasoning and independent work.
   - It develops initiative and self-responsibility among the students.
   - It takes into account individual differences.
   - It helps the students to develop reflective thinking.
   - It helps the students to approach future problems with confidence.
   - It builds a mental attitude for effective learning based on critical thinking.
   - It helps the children develop mental traits of open-mindedness and tolerance as the children see many sides to a problem and listen to many points of view.

6. Teacher plays a significant role in problem solving method. The teacher's role is to:

   - Ensure an atmosphere of freedom in the class.
   - Create the problem situation.

- Help the students in accepting, defining and stating the problem.
- Help the students in analysing the problem and in breaking up the problem into simple units.
- Help the students keep their attention focussed on the main problem all the time.
- Assist the students in locating relevant source materials.

7. Inductive Method is based on induction. Induction is the process of proving a universal truth or a theorem by showing that if it is true of any particular case, it is true of the next case in the same serial order and hence true for any such cases. So the technique of making transition from particular facts to generalisations about these facts is known as the process of induction. Thus it is a method of arriving at a formula or a rule by observing a sufficient number of particular instances. If one rule applies to a particular case and is equally applicable to different similar cases, it is accepted as a general rule or formula. Therefore, in this method we proceed from particular to general, from concrete instances to abstract rules and from simple examples to complex formula. A formula or a generalisation is arrived at through inductive reasoning.
8. Merits of Inductive Method
   - It helps understanding.
   - It is a logical method and develops critical thinking.
   - It encourages active participation of the students in learning.
   - It provides ample opportunities for exploration and observation.
9. Steps in Deductive Method

   Deductive method of teaching follows the steps given below.

   - Clear recognition of the problem: A clear recognition of the problem statement provides the basic link for the thinking process and the solution to the problem.
   - Search for a tentative hypothesis: The second step in deductive method is the search for a tentative hypothesis, a tentative solution to the problem.
   - Formulation of a tentative hypothesis: The search for the solution leads to the formulation of a tentative hypothesis that appears to have promise as a possible or probable solution to the problem. The tentative hypothesis has its basis on certain

axioms or postulates, or propositions or rules and formulae that have been accepted to be true.

- Verification: Finally the hypothesis that has been formulated is to be verified as the right solution to the problem at hand.

10. Creative teaching is influenced by various components. There are at least six components.

The **first component** is related to basic pedagogical skills such as lesson planning, classroom management, communication, and evaluation.

The **second component** refers to the domain specific expertise, creative techniques, and knowledge of developmental processes.

The **third component** is related to the competence in selecting appropriate assessment modes.

The **fourth component** refers to teachers' and students' motivation. Intrinsic motivation is an indispensable component as it generates an on-going commitment.

The **fifth and the sixth components** are related to the learning climate and environment: Educational policies and the school culture. Creative mathematics teaching is influenced by educational policies and school cultures. Educational policies influence school learning climates.

6.11 SUGGESTED READINGS

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NOTES

UNIT – VII
LEARNING RESOURCES AND MATERIAL PRODUCTION - I

Structure

7.1 INTRODUCTION

A teacher has an inherent desire that his teaching should be as effective as possible. Effective teaching in mathematics depends upon many factors. The arrangement and equipment of the mathematics classroom is one such important factor. Mathematics Library is also an important source of acquiring mathematical knowledge and skills. The mathematics teacher should motivate the students to use the Library more productively. The learning can be made easier and effective if the teaching is supplemented with the use of different types of aid materials and resources such as models, concrete objects, apparatuses, instruments, interactive board, geo board, low – cost teaching materials and other resources.

This unit presents the importance of mathematics classroom, Need and Functions of a Good Mathematics Library, Materials and Equipments for a Mathematics Laboratory, Functions of a mathematics laboratory and the importance of Low cost teaching aids for Teaching Mathematics.

7.2 OBJECTIVES

At the end of the unit, the student-teachers will be able to

- State the favourable conditions of a mathematics classroom.
- Describe the necessity of mathematics library and laboratory for teaching.

- Create a range of learning opportunities for students with the help of mathematics library and laboratory.
- Explain how the low-cost teaching materials can heighten the effect of teaching mathematics.
- Know how to use the low-cost teaching materials in their mathematics teaching.

7.3 MATHEMATICS CLASSROOM

Effective instruction in Mathematics depends upon many factors. The arrangement and equipment of the Mathematics classroom is one such important factor. The class-room should be equipped in such a way that it will facilitate proper instruction.

The classroom should have a large black board and graph board. It should have black board instruments like protractors, compasses, dividers, setsquares, parallel rulers, meter sticks and meter scales. There can be display case containing models of geometrical solids, diagrams of geometrical figures and models98. Photographs and pictures of great Mathematicians can be displayed on the walls.

The room should be equipped with a demonstration table for setting up experimental apparatus. It can have a work table where small groups of students may work together. The importance of such a work table will be felt when the laboratory method is adopted. Pictures relating to the history, progress and application of Mathematics will heighten the attractiveness of the class-room. The desk top should be large enough to facilitate pupils to write, draw and use instruments.

Such a room is sure to create a favorable climate for stimulating interest in the learning of Mathematics.

7.4 MATHEMATICS LIBRARY

Mathematics library is one of the important sources of acquiring mathematical knowledge and skills. It serves the purpose of inculcating good reading and study habits which help in promoting self-study and independent work among students. The students, at a young age should be exposed to the joy of reading and should be equipped with adequate reference skills. However, with the advent of information technology and cyber cafes, there is a growing decline in the reading of books. The different types of books available in the mathematics library help the students in promoting reading habits.

Need and Functions of a Good Mathematics Library

Even though there is a general library with a separate section for Mathematics Department in every institution, it is necessary to have a separate library for mathematics for the following reasons.

- A mathematics library provides immediate access to the necessary mathematics books
- It facilitates timely use of the books and inculcates reference skills. As the mathematics teacher is in charge of the mathematics library, the students can seek the teacher's help and guidance in locating and selecting the relevant books.
- It helps the mathematics teacher to get acquainted with the latest developments in mathematics and innovative techniques of teaching, as the teacher is constantly in touch with the new additions to the library.
- It provides the mathematics teacher with an important role in the selection of relevant books for the mathematics library.
- It promotes efficiency in the organisation of library services.
- It provides a sense of separate identity to mathematics and helps to motivate the students to learn mathematics with genuine interest.

The mathematics teacher should take the responsibility of organising the mathematics library and be in charge of such a library. Mathematics library can be situated in a large classroom or in a separate section in the mathematics laboratory. The effective use of the library by the students can be ensured by allotting a separate period in the timetable for library work. The mathematics teacher should motivate the students to use the library time more productively.

A mathematics library serves the following educational purposes

- A mathematics library provides the extra information necessary for mathematics, since a teacher of mathematics may not find time to go beyond the prescribed textbooks.
- Mathematics learning requires a lot of drill and practice, a particular set of textbooks cannot serve this purpose. A mathematics library helps in drill and practice as the students can refer a number of books other than the prescribed textbooks.
- The different types of problems and approaches available in different books help the students in developing their problem-solving ability and adopting different approaches to problem-solving, thus making them more open-minded and independent.
- Classroom teaching may leave many gaps and doubts unclarified.
  A good library provides opportunities for filling up the gaps and removing the doubts.

- It provides facilities for the gifted students to satisfy their curiosity and thirst for knowledge.
- It provides adequate materials and information necessary for completing the assignments and homework.
- It helps in developing reference skills necessary for higher education.

Books and Materials for a Mathematics Library

The mathematics library should be attractive and should provide an atmosphere conducive for reading. It can be decorated with visual aids and pictures of mathematicians, charts and posters about their contributions, information on historical development of certain mathematical concepts, theorems and so on. The seating arrangement must be comfortable for the students to sit and read. Above all, the books selected for a mathematics library should be relevant, interesting, useful and should enthuse the students to read. While selecting the books, the emphasis should not be merely on quantity, but also on quality. The books and materials for a mathematics library can be collected and categorised under the following heads and can be arranged in different cabins, bureaus and almirhas.

- The prescribed text books of mathematics for the different classes.
- Books of mathematics at a higher levels for references for teachers and students.
- Books on Methods and Techniques of Teaching (These are reference books for teachers to acquaint them with innovative methods and techniques of teaching mathematics.

Mathematics Journals

The mathematics library must have journals to acquaint the teachers and students with the current events and research findings in mathematics.

- The Mathematics Education (A quarterly journal)
- The Mathematics Teacher (A bimonthly journal)
- Periodicals and Newsletters published by NCERT and SCERT from time to time

Other Useful Literature

- Reports of the various committees and commissions on mathematics education.
- Information on career guidance relating to mathematics education

7.5 Mathematics Laboratory

It is true that the mathematics laboratory has not yet received the same general acceptance as a science laboratory. This is probably because the mathematics teachers themselves have not recognized the significance of mathematics laboratory as the science teachers have. Actually most mathematical teachers have been very passive as to this respect. A few years back, most of mathematics teachers did not visualise a concrete mathematics laboratory. But, now a days, having at least a mini Laboratory in schools has gained proficiency. Let us see, importance of Mathematics laboratory.

- Combines theory with practice.
- Develops scientific outlook.
- Satisfies different needs of students.
- Constructive eagerness and creative energy of child gets proper outlet.
- Variety of methods are offered by laboratory.
- It has everlasting positive impact on students.
- Makes students research minded.
- It is multi-sensory way of learning.
- Develops love for the subject.
- Attitude of attacking the problem directly, is developed in students.

Materials and Equipments for a Mathematics Laboratory

For the effective functioning of a mathematics laboratory, it should be well equipped. A mathematics laboratory may contain the following type of materials and equipments.

Materials

- **Concrete Materials**

The mathematics laboratory should contain materials such as beads, pebbles, sticks, ball frames, seeds, balance, weights, measuring tapes, scissors, pins, abacus, cardboard, board pins, chart paper, graph paper etc. which are very helpful for demonstrating elementary mathematical concepts

- **Pictures and Charts**

Pictures and charts depicting different mathematical concepts should find a place in a mathematics laboratory. Pictures of mathematicians, charts showing the contributions of mathematicians,

history of mathematics, biographies of mathematicians are also very helpful in inspiring students and can be placed in the mathematics laboratory.

- **Models**

Various mathematical models which will offer opportunities for students to explore and investigate should be placed in the mathematical laboratory.

Bulletin Board

There should be at least one bulletin board in the mathematics laboratory to display various illustrations concerning mathematics. Information relating to mathematics and its applications collected from different sources like magazines, journals and newspapers can be displayed for the benefit of all the pupils.

Blackboard

A blackboard should be provided in the mathematics laboratory. This helps in writing instructions, drawing geometrical figures and illustrations necessary for performing the laboratory work.

Instruments and Equipments

Drawing Instruments

In mathematics laboratory, the students are required to draw and sketch geometrical figures, graphs and so on. Therefore, in every mathematics laboratory there should be a set of drawing instruments like compass, rulers, protractor etc. for the teachers to demonstrate the drawing on the blackboard and instrument boxes for the students' use. Stencils for drawing geometrical figures are also important tools in a mathematics laboratory.

Weighing and Measuring Instruments

Mathematical laboratory work, quite often, involves the important activities of weighing and measuring. Therefore, instruments used for measuring and weighing such as measuring tapes, balances of different types, measuring jar, and graduated cylinder should be there in a mathematics laboratory.

Surveying Instruments

Surveying is an important aspect of mathematical laboratory work. Laying out right angles, finding the angular distance, estimating the height of a building, estimating the distance of an object, finding the angle of elevation and depression are all useful activities which make use of surveying instruments. The following instruments are very useful in this regard and they should have a place in the mathematics laboratory.

- Angle mirror for laying out right angles in the field.

- Sextant for measuring i) the angular distance
  ii) angles of elevation and depression
  iii) the height of an object
  iv) the width of a river etc.
- Plane table and Alidote for elementary mapping and surveying
- Clinometers for estimating the i) angle of elevation
  ii) height of an object
- Transit for measuring the angular distance of two objects
- Level for finding differences in elevation.

Proportional Dividers

This instrument works on the principle of proportionality of similar triangles. It can be used for enlarging or reducing figures, pictures, maps, graphs etc.

Computing Devices

Slide rules and calculators

The students need to do the computation with speed and accuracy. In order to achieve this, a mathematics laboratory should have calculating devices like slide rules and calculators. A slide rule consists of two or more logarithmic scales sliding on each other. It provides rapid means of arithmetic operations and is quite handy in those schools which cannot afford to buy calculators. Calculators are very helpful in doing complex computations with greater speed and accuracy.

Use of calculators

An advantage of calculators over computers is that they are smaller in size and lend themselves to systematic use over a long period. If this is to be effective, then they must be given the opportunity to do more enjoyable and productive mathematics. These days calculators have become very common among schoolchildren and they use them, quite frequently for computation. But the systematic use of calculators should be explicitly planned for by the teacher. Inappropriate use of calculators, for example, where mental calculation would be possible and more desirable, can quickly lead to more endemic situations where their only use is button pressing for thinking. In this sense, then the uncritical use of calculators can be counterproductive.

Computer

In this technological age one cannot overestimate the role of computers in the mathematical work. A computer is an effective device for storing information; analysing and interpreting data. Therefore, it is

desirable to have a computer in the mathematics laboratory to facilitate easy functioning of the mathematical laboratory.

The list of materials listed above is not exhaustive A teacher of mathematics can use his discretion in selecting the instruments to equip the mathematics laboratory. The type of instruments and materials for a mathematics laboratory, to a great extent, depends upon the needs of the students and the nature of the mathematical activities carried out in the mathematics laboratory.

Functions of a mathematics laboratory

The primary functions of a mathematics laboratory are to

- make mathematics teaching and learning, interesting and purposeful for the students
- provide activities that arouse the curiosity of the students and maintain their interest in learning.
- enable students to develop proper skills in handling equipment and gadgets.
- make students appreciate the practical applicability of mathematical principles and laws
- concretise the abstract mathematical concepts
- help the students develop powers of observation, analysis and drawing inferences.

Check Your Progress

Notes : a) Write your answers in the space given below

b) Compare your answer with the one given at the end of the unit

1. Write a short note on Mathematics classroom.

   ……

   ……

2. Why a Good Mathematics Library is needed for a school?

   ……

   ……

3. List out the Functions of a mathematics laboratory

   ……

   ……

7.6 PREPARATION OF LOW COST TEACHING AIDS FOR TEACHING MATHEMATICS

Mathematics is a type of subject the need of which is felt well for performing the day to day activities of our life. Not only this a child from his tender age observes the use and application of mathematics in the

existence and activities of all the animal and inanimate objects surrounding him. Infact nature itself is a big source and a treasurer of aid material for learning the facts and principles of mathematics. The children may thus be helped to gather valuable direct and indirect experiences for the learning of mathematics from their local environmental surroundings. In their local set up comprising of their homes, community, physical and social environment, they may get a lot of opportunity to practice and learn so many valuable concepts regarding the teaching learning of mathematics. There is a lot of cheap and sometimes waste material available in children's local environment that can be successfully utilized for the improvisation of valuable aid material for the teaching of Mathematics. Let us see what type of aid material can be easily improvised for the teaching and learning of Mathematics.

1. **Concrete material and objects.** A wide variety of collection of different types of concrete material like beads, seeds, balls, sticks, match boxes, pebbles, different types of corns etc. may be made with the help of students. All that material may prove quite helpful in the learning of counting, four fundamental rules (addition, subtraction, multiplication, and division), multiplication tables etc.

2. **Improvising an abacus.** An Abacus containing a number of wood, metal or even thermocol beads in several wires can be easily improvised for teaching the students the facts of counting, four fundamental rules, place value systems etc.

3. **Place value pockets** may also be improvised. The required boxes for this purpose can be made with the help of thermocol, thick paper or wooden etc. and the system be made as operated as help in the learning of the concept of place value.

4. **Preparation of Models.** By using the easily available low cost or waste material various types of models may be improvised. Let us have different types of such models representing the shape of various geometrical figures like rectangle, square, parallelogram, trapezium, triangle, circle, ellipse, cone, cylinder, pyramid, sphere etc. We can have models made of clay, match box, sticks, thermocol, wood, wax etc. To demonstrate the various geometrical figures, their properties and operations. Area of the cross roads, area of the four walls of a room, circumference and area of a circle, circumference and area of rectangle, triangle and so many other things related to the learning of menstruation may be easily taught through the use of the models made of the locally available low cost material. The waste paper, card boards, thick papers, wooden boxes etc. Available in the

packages of the household goods purchased from market can be effectively used for making that model with a simple use of pins, nails, scissors, hammers, threads, ropes etc. Easily available in the houses and school workshop. For illustration purpose let me point out a working model for helping the students to understand geometrical concept and the related theorem "Sum of the three angles of a triangle is equal to two right angles." It has been already mentioned in the present chapter. We may utilize the wood available from the carton boxes or the boxes received from shopkeepers during the purpose of fruits and other household articles for the improvisation of this model.

5. **Preparation of Charts and Pictures.** With the help of the chart paper and drawing material easily available in the market, the low cost visual aid material may be easily prepared for the teaching, learning of almost all concepts related to all the branches of mathematics. You may hang these charts in the mathematics classrooms, laboratory or library for the grasping of the essential mathematical concepts with no extra efforts. At the time of class room teaching, these charts and pictures may be successfully used for the clarity of the needed mathematical concepts, processes and operations.

A teacher of mathematics in this way may utilize the waste of low cost material available in the local surroundings for the preparation and improvising valuable teaching – learning aids for bringing efficiency and effectiveness to the ongoing teaching learning process. He may even utilize the real surroundings as an aid to his teaching. For example, while teaching about the area of the four walls of a room, he can have the class room as a living concrete model. In teaching the concept of mensuration, than he may utilize the sports ground, school, garden and neighboring plots as a living models for necessary surveying and measurement. The scrap book prepared by the students through the news paper cuttings of the data of mathematical interest thus may also be used as one of the useful aid material for acquainting the students with the facts and principles of mathematics. As a matter of conclusion, thus teacher of mathematics should always try to exploit his resourcefulness, creativity and ingenuity for making the task of teaching-learning interesting and purposeful by trying to make use of the low cost improvised teaching aids as and when he finds opportunities to do so with the help of his students.

Process of developing Low-Cost Teaching Aids:

The different steps involved in are:

(i) Defining of objects. The objective of the teaching aid in terms of knowledge, skills and attitudes to be developed are defined in the light of the needs of the users.

(ii) Preparation of a design. A design for the development of different aids to be developed, its cost, relevance and the resources available in the local environment.

(iii) Development of Aids. After having decided the design, the aids are developed by students, teachers, specialists or community in cooperation with each other.

(iv) Pilot testing of the Aids. The pilot testing of the aids is done by the teachers or researchers with selected sample users. On the basis of the results of pilot testing necessary improvements are made in the aids. This also provides a feedback for modifying objectives and design of the aid if necessary.

(v) Finalization of Aids. If the aid is considered satisfactory after pilot-testing.

(vi) Production and distribution. Adequate numbers of copies of the final aids are produced. It is distributed to different schools if it is considered valuable to users.

Check Your Progress

Notes : a) Write your answers in the space given below

b) Compare your answer with the one given at the end of the unit

4. What are the steps involved in the process of developing low-cost teaching aids?

……

……

5. Write a short note on Preparation of Charts and Pictures.

……

……

7.7 LET US SUM UP

This unit attempted to deal with learning resources and material production. Effective instruction in Mathematics depends upon many factors. The arrangement and equipment of the Mathematics classroom is one such important factor. The class-room should be equipped in such a way that it will facilitate proper instruction.

Mathematics library is one of the important sources of acquiring mathematical knowledge and skills. It serves the purpose of inculcating good reading and study habits which help in promoting self-study and independent work among students.

Web based learning is often called online learning or e-learning because it includes online course content. The current focus of WBL development is on learning how to use the available tools and organize content into well- crafted teaching systems.

Interactive whiteboards are used in lecture or classroom environments and the technology allows you to write or draw on the surface, print off the image, save it to computer or distribute it over a network. They make it easy for teachers to enhance presentation content by easily integrating a wide range of material into a lesson, such as a picture from the internet, a graph from a spreadsheet or text from a Microsoft word file, in addition to student and teacher annotations on these objects.

A **geoboard** is a mathematical manipulative used to explore basic concepts in plane geometry such as perimeter, area and the characteristics of triangles and other polygons.

The *Geometer's Sketchpad* is the world's leading software for teaching mathematics. Sketchpad is the optimal tool for interactive whiteboards. Teachers can use it daily to illustrate and illuminate mathematical ideas.

7.8 UNIT-END ACTIVITIES

1) Justify the need of mathematics laboratory for effective teaching of mathematics.
2) What are the conditions required for a classroom that will facilitate the learning of mathematics?
3) Mathematics library is another neglected aspect of teaching mathematics. How will you proceed to remedy the existing conditions?
4) Give your views regarding the preparation of low cost teaching aids in the subject mathematics.

7.9 ANSWERS TO CHECK YOUR PROGRESS

1. Effective instruction in Mathematics depends upon many factors. The arrangement and equipment of the Mathematics classroom is one such important factor. The class-room should be equipped in such a way that it will facilitate proper instruction. The class-room should have a large black board and graph board. It should have black board instruments like protractors, compasses, dividers, setsquares, parallel rulers, meter sticks and meter scales. There can be display case containing models of geometrical solids, diagrams of geometrical figures and models 98. Photographs and pictures of great Mathematicians can be displayed on the walls.

2. Need and Functions of a Good Mathematics Library

   - A mathematics library provides immediate access to the necessary mathematics books
   - It facilitates timely use of the books and inculcates reference skills. As the mathematics teacher is in charge of the mathematics library, the students can seek the teacher's help and guidance in locating and selecting the relevant books.
   - It helps the mathematics teacher to get acquainted with the latest developments in mathematics and innovative techniques of teaching, as the teacher is constantly in touch with the new additions to the library.
   - It provides the mathematics teacher with an important role in the selection of relevant books for the mathematics library.
   - It promotes efficiency in the organisation of library services.
   - It provides a sense of separate identity to mathematics and helps to motivate the students to learn mathematics with genuine interest.

3. The primary functions of a mathematics laboratory are to

   - make mathematics teaching and learning, interesting and purposeful for the students
   - provide activities that arouse the curiosity of the students and maintain their interest in learning.
   - enable students to develop proper skills in handling equipment and gadgets.
   - make students appreciate the practical applicability of mathematical principles and laws
   - concretise the abstract mathematical concepts
   - help the students develop powers of observation, analysis and drawing inferences.

4. Steps involved in the process of developing Low-Cost Teaching Aids

   a. Defining of objects.
   b. Preparation of a design.
   c. Development of Aids.
   d. Pilot testing of the Aids.

e. Finalization of Aids.
f. Production and distribution

5. With the help of the chart paper and drawing material easily available in the market, the low cost visual aid material may be easily prepared for the teaching, learning of almost all concepts related to all the branches of mathematics.

7.10 SUGGESTED READINGS

- Anice James (2011). Teaching of Mathematics; Neelkamal publications, Hyderabad.
- Dececco J.P., Educational Technologies – Readings in Programmed Instruction, Holt, New York.
- Kulbir Singh Sidhu(2006) The Teaching of Mathematics; Sterling Publishers, New Delhi.
- Mangal, S.K(2005). Teaching of Mathematics; Tandon Publications, Ludhiana.
- Sonia Bhasin(2005). Teaching of Mathematics – A Practical Approach; Himalaya Publishing House, Mumbai.
- SudhirKukar, (1993) Teaching of Mathematics; AnmolPublishers,New Delhi.
- Wangoo, M.L., (2002) Teaching of Mathematics; Bharat Publications, Ludhiana.

UNIT – VIII

LEARNING RESOURCES AND MATERIAL PRODUCTION - II

Structure

8.1 INTRODUCTION

Web-based learning is one way to learn, using web-based technologies or tools in a learning process. The current focus of **Web-based learning** development is on learning how to use the available tools and organize content into well- crafted teaching systems. Interactive whiteboards create a range of learning opportunities for both students and teachers. Interactive whiteboards are used in lecture or classroom environments and the technology allows a teacher to write or draw on the surface, print off the image, save it to computer or distribute it over a network.

A geoboard is a mathematical manipulative used to explore basic concepts in plane geometry such as perimeter, area and the characteristics of triangles and other polygons.

The Geometer's Sketchpad is a popular commercial interactive geometry software program for exploring Euclidean geometry, algebra, calculus, and other areas of mathematics.

This present unit presents all such materials, resources and technologies which will help the mathematics teachers for making their teaching more effective and interesting.

8.2 OBJECTIVES

At the end of the unit, the student-teachers will be able to

- Know about **web-based technologies**.
- Know how to use the available web-based technologies and tools in their mathematics teaching.
- Describe the necessity of using geoboard and Sketchpad for teaching mathematics.
- Explain how the web-based technologies and tools can heighten the effect of teaching mathematics.
- Create a range of learning opportunities for students with the help of interactive board, geo board, sketchpad etc.

8.3 WEB BASED LEARNING

Web-based learning is one way to learn, using web-based technologies or tools in a learning process. In other words, learner uses mainly computers to interact with the teacher, other students and learning material. **Web-based learning consists of technology that supports traditional classroom training and online learning environments.**

Web based learning is often called online learning or e-learning because it includes online course content. Discussion forums via email, videoconferencing, and live lectures (video streaming) are all possible through the web. Web based courses may also provide static pages such as printed course materials. One of the values of using the web to access course materials is that web pages may contain hyperlinks to other parts of the web, thus enabling access to a vast amount of web based information. The current focus of WBL development is on learning how to use the available tools and organize content into well- crafted teaching systems.

Advantages and Disadvantages of Web-based learning

Web-based learning has both advantages and disadvantages. When comparing them, we can notice that the same factors can be advantages as well as disadvantages depending on the context. Below some factors are described.

Factor	Advantages	Disadvantages
Learning theories and approaches	New learning theories and approaches enable to learn and teach in a more effective way. Students can experience a sense of equality. Course work and challenging assignments are stimulating for knowledge building.	Teachers and learners have to adopt new learning theories and approaches. Role changes of teachers and learners may cause frustration and confusion. Without the common structures of a traditional class, students may feel

		lost or confused about course activities and deadlines.
Independent and learner centred learning	Students can work at their own pace, when they want. Web-based learning enables to study more deeply areas of interest. It encourages exploring material on your own and enables to skip over materials already mastered. Web-based learning supports personalised learning and is self-directed. It builds self-knowledge and self-confidence and encourages students to take responsibility for their own learning.	Learners who are not self-motivated, self-directed and independent are not able to plan their own learning and may have problems. Material and assignment instructions might be too complicated to understand independently. Some of the students may lose motivation without certain deadlines. Independent learning requires certain skills: technological, communication skills, self-motivation and effective study habits.
Flexibility	Web-based learning enables to join discussions at any hour and encourages also those who don't like to speak. It facilitates learning through a variety of activities. Learners have access to courses, which enables to reduce travel time and costs.	Learners with low motivation or bad study habits may fall behind. They may have difficulties in organizing their learning.
Interaction	Web-based learning provides interaction between students and instructors. Students can share their ideas with other students, which may help to understand the material better.	Prohibits those who are not active learners in a group. Human contact is missing as interaction is relied on electronic communication.
Access to material	Students can study anywhere and anytime if they have an access to computer and Internet. Web-based learning	Problems with technology might prevent the access: low speed connection, difficulties to download information, problems

	provides continual and also direct access to materials, resources in many different formats and of good quality.	with communication tools. Some courses and materials might be out of date. There may be lack of quality control.
Contact with instructors	Working on the web offers an opportunity to communicate with students using e-mail, discussion boards etc. Teachers receive students' work quickly and they provide timely feedback to students' questions.	Teachers are overloaded with students and their contacts. Students may feel isolated from the instructor and classmates. Instructor may not always be available when students are studying or need help.
Use of technology	When you learn to use one browser and certain software, you will probably be able to use other browsers and software as well. Some of the software and web browsers are free of charge on the internet. Web-based learning develops knowledge of the Internet and computer skills that help learners throughout their lives and careers.	Managing computer files and online learning software can be complex for students with beginner-level computer skills. Poor usability may cause troubles with navigation, computers crash or have viruses, impossible to send mails. Software and access to the Internet and e-mail is not free all the time

8.4 INTERACTIVE WHITEBOARD

Interactive whiteboards create a range of learning opportunities for both students and teachers. Interactive whiteboards are used in lecture or classroom environments and the technology allows a teacher to write or draw on the surface, print off the image, save it to computer or distribute it over a network. The teacher can also project a computer screen image on to the surface of the whiteboard and then either control the application by touching the board directly or by using a special pen. The computer image can be annotated or drawn over and the annotations saved to disc or sent by email to others.

What are the benefits?

- Because interactive whiteboards are so like conventional whiteboards, they can help even technophobic teachers to use

this medium with ease for presentations from the front of the room.

- They help in broadening the use of e-learning because they rapidly demonstrate the potential of alternative modes of delivery.
- They make it easy for teachers to enhance presentation content by easily integrating a wide range of material into a lesson, such as a picture from the internet, a graph from a spreadsheet or text from a Microsoft word file, in addition to student and teacher annotations on these objects.
- They allow teachers to create easily and rapidly customized learning objects from a range of existing content and to adapt it to the needs of the class in real time.
- They allow learners to absorb information more easily.
- They allow learners to participate in group discussions by freeing them from note – taking.
- They allow learners to work collaboratively around a shared task or work area.
- When fully integrated into a VLE (Virtual Learning Environment) and learning object repository there is potential for widespread sharing of resources.
- When used for interactive testing of understanding for the entire class, they can rapidly provide learner feedback.

What are the disadvantages?

- Interactive whiteboards are more expensive than conventional whiteboards or projector and screen combinations.
- Their surface can become damaged, necessitating expensive replacement.
- Front projection boards are often mounted too high for users to reach the top of or too low to be readily visible by all users.
- Free-standing boards (and their associated projectors) are more difficult to secure and need to be realigned every time they are moved.
- If multiple data entry is allowed, inputs can get jumbled, resulting in on-screen gibberish.
- If remote access is allowed, some users may be tempted to send disruptive comments or drawings to the screen.

Check Your Progress

Notes : a) Write your answers in the space given below

b) Compare your answer with the one given at the end of the unit

1. Write a short note on web based learning.

   ……

   ……

2. Mention any two advantages of web based learning.

   ……

   ……

3. What are the benefits of Interactive whiteboards?

   ……

   ……

8.5 GEOBOARD

A geoboard is a mathematical manipulative used to explore basic concepts in plane geometry such as perimeter, area and the characteristics of triangles and other polygons. It consists of a physical board with a certain number of nails half driven in, around which are wrapped rubber bands. Geoboards are grids of pegs that can hold rubber bands in position. Geoboards were invented and popularized in the 1950s by Egyptian mathematician Caleb Gattegno (1911-1988).

Geoboards are now available in a variety of sizes, styles, and colours.

A preferred model is the transparent geoboard that can be placed on an overhead projector to facilitate sharing of student observations and conclusions. Many geoboard activities are available for both the 5 pin x 5 pin and 11 pin x 11 pin geoboard sizes.

Some Sample Activities

1. Construct a design for a quilt square or a stained glass window. Analyze the design. Enlarge or reduce the design. Compare your design with another student's design. (What's the same? What's different?)
2. Construct two three-sided (or four-sided) figures that are congruent (or similar).
3. Determine how many different sizes of squares (or equilateral triangles) can be constructed on a 5-pin x 5-pin geoboard. How many can be constructed on a 20-pin by 20-pin geoboard?

4. Construct two pentagons that have the same area but different perimeter (or vice-versa).
5. Construct a symmetrical design.
6. Construct a "diagonal" segment. Now construct a line segment that is parallel to the first segment. Determine the lengths and slopes of both segments.
7. Construct a line segment whose length has a measure between 3 and 4 units (or a given slope).
8. Choose any two pegs on the geoboard and determine different paths from the first peg to the second. Record solutions and determine which path is the longest and which is the shortest.
9. Create a 3-sided (or 4-sided) shape. Compare with a partner. What is the same? What is different?
10. Create a shape. Determine its area in more than one way.
11. Design a shape on one half of the geoboard. Construct its reflection.
12. Make a figure that has an area of 4 and a perimeter of 10.
13. Determine the maximum area that can be enclosed on an 11-pin by 11-pin geoboard if the perimeter is 25 units.
14. Determine different ways to divide the geoboard into 4 equal areas.
15. Use rubber bands to divide the geoboard into different areas. Express each area as a fraction (decimal, percent) of the whole area.

8.6 SKETCHPAD

The *Geometer's Sketchpad* is the world's leading software for teaching mathematics. *Sketchpad* gives students at all levels—from primary through college—a tangible, visual way to learn mathematics that increases their engagement, understanding, and achievement. Make math more meaningful and memorable using Sketchpad.

Elementary students can manipulate dynamic models of fractions, number lines, and geometric patterns. Middle school students can build their readiness for algebra by exploring ratio and proportion, rate of change, and functional relationships through numeric, tabular, and graphical representations. And high school students can use Sketchpad to construct and transform geometric shapes and functions—from linear to trigonometric—promoting deep understanding. Sketchpad is the optimal tool for interactive whiteboards. Teachers can use it daily to illustrate and illuminate mathematical ideas. Classroom-

tested activities are accompanied by presentation sketches and detailed teacher notes, which provide suggestions for use by teachers as a demonstration tool or for use by students in a computer lab or on laptops.

Geometer's Sketchpad Briefly, upon activating the Sketchpad program, a drawing window will be opened with the following: Title bar Menu bar Tool Box (or Drawing Tools) Tool Status Box Then what we can do with Sketchpad? We can do seven major tasks:

1. Euclidean constructions
2. Transformations
3. Analytic Geometry – working with rectangular or polar coordinates
4. Mix Graphs and Text
5. Change the Visual Properties of the displayed Figures
6. Create Animations
7. Creating Scripts (with which a complex construction can be encapsulated in a single step).

The Geometer's Sketchpad is a popular commercial interactive geometry software program for exploring Euclidean geometry,algebra, calculus, and other areas of mathematics. Geometer's Sketchpad includes the traditional Euclidean tools of classical geometric constructions; that is, if a figure (such as the pentadecagon) can be constructed with compass and straight-edge, it can also be constructed using this program. Objects can also be animated. The program also allows the determination of the midpoint and midsegments of objects.

Geometer's Sketchpad also allows one to measure lengths of segments, measures of angles, area, perimeter, etc. Some of the tools one can use include construct function, which allows the user to create objects in relation to selected objects. The transform function allows the user to create points in relation to objects, which include distance, angle, ratio, and others. With these tools, one can create numerous different objects, measure them, and potentially figure out hard-to-solve math problems.

Educational Benefits

Geometer's Sketchpad is used in many secondary mathematics classrooms. NCTM (National Council of Teachers of Mathematics) had identified one of its six principles as a technology principle, stating that "Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning." Geometer's Sketchpad is one of these examples. The program comes with program files to help deepen students' understanding of such concepts as slope, geometric transformations, and arithmetic on integers. Geometer's Sketchpad allows students to construct precise figures and

manipulate them interactively. It helps them to develop mental models for thinking about geometric shapes and their properties. Geometer's Sketchpad is ideal for cooperative learning but also serves teachers well as a demonstration tool so that if the instructor has a limited amount of computers, students can still understand the visual aspects of Geometry.

Check Your Progress

Notes : a) Write your answers in the space given below

b) Compare your answer with the one given at the end of the unit

4. What is Geoboard?

……

……

5. Write a short note on Geometer's Sketchpad.

……

……

6. What are the educational benefits of Geometer's Sketchpad?

……

……

8.7 LET US SUM UP

Web based learning is often called online learning or e-learning because it includes online course content. The current focus of WBL development is on learning how to use the available tools and organize content into well- crafted teaching systems.

Interactive whiteboards are used in lecture or classroom environments and the technology allows you to write or draw on the surface, print off the image, save it to computer or distribute it over a network. They make it easy for teachers to enhance presentation content by easily integrating a wide range of material into a lesson, such as a picture from the internet, a graph from a spreadsheet or text from a Microsoft word file, in addition to student and teacher annotations on these objects.

A geoboard is a mathematical manipulative used to explore basic concepts in plane geometry such as perimeter, area and the characteristics of triangles and other polygons.

The *Geometer's Sketchpad* is the world's leading software for teaching mathematics. Sketchpad is the optimal tool for interactive whiteboards. Teachers can use it daily to illustrate and illuminate mathematical ideas.

8.8 UNIT-END ACTIVITIES

5) Justify the need of mathematics laboratory for effective teaching of mathematics.

6) What are the conditions required for a classroom that will facilitate the learning of mathematics?

7) Mathematics library is another neglected aspect of teaching mathematics. How will you proceed to remedy the existing conditions?

8) Give your views regarding the preparation of low cost teaching aids in the subject mathematics.

9) What is interactive whiteboard? Throw light on their use in the teaching and learning of mathematics.

10) Explain the use and importance of the following

    a) Geo Board.
    b) Sketch Pad.

8.9 ANSWERS TO CHECK YOUR PROGRESS

1. **Web-based learning is one way to learn, using web-based technologies or tools in a learning process.** In other words, learner uses mainly computers to interact with the teacher, other students and learning material. **Web-based learning consists of technology that supports traditional classroom training and online learning environments.** Web based learning is often called online learning or e-learning because it includes online course content.

2. Students can work at their own pace, when they want. Web-based learning enables to study more deeply areas of interest.

3. The benefits of Interactive whiteboards

    - Because interactive whiteboards are so like conventional whiteboards, they can help even technophobic teachers to use this medium with ease for presentations from the front of the room.
    - They help in broadening the use of e-learning because they rapidly demonstrate the potential of alternative modes of delivery.
    - They allow teachers to create easily and rapidly customized learning objects from a range of existing content and to adapt it to the needs of the class in real time.
    - They allow learners to absorb information more easily.

4. A geoboard is a mathematical manipulative used to explore basic concepts in plane geometry such as perimeter, area and the characteristics of triangles and other polygons. It consists of a physical board with a certain number of nails half driven in, around which are wrapped rubber bands.
5. The Geometer's Sketchpad is a popular commercial interactive geometry software program for exploring Euclidean geometry, algebra, calculus, and other areas of mathematics. Geometer's Sketchpad includes the traditional Euclidean tools of classical geometric constructions; that is, if a figure (such as the pentadecagon) can be constructed with compass and straight-edge, it can also be constructed using this program. Objects can also be animated. The program also allows the determination of the midpoint and midsegments of objects.
6. Educational Benefits of Geometer's Sketchpad is used in many secondary mathematics classrooms. Geometer's Sketchpad allows students to construct precise figures and manipulate them interactively. It helps them to develop mental models for thinking about geometric shapes and their properties. Geometer's Sketchpad is ideal for cooperative learning but also serves teachers well as a demonstration tool so that if the instructor has a limited amount of computers, students can still understand the visual aspects of Geometry.

8.10 SUGGESTED READINGS

- Anice James (2011). Teaching of Mathematics; Neelkamal publications, Hyderabad.
- Dececco J.P., Educational Technologies – Readings in Programmed Instruction, Holt, New York.
- Kulbir Singh Sidhu(2006) The Teaching of Mathematics; Sterling Publishers, New Delhi.
- Mangal, S.K(2005). Teaching of Mathematics; Tandon Publications, Ludhiana.
- Sonia Bhasin(2005). Teaching of Mathematics – A Practical Approach; Himalaya Publishing House, Mumbai.
- SudhirKukar, (1993) Teaching of Mathematics; AnmolPublishers,New Delhi.
- Wangoo, M.L., (2002) Teaching of Mathematics; Bharat Publications, Ludhiana.

UNIT IX
PLANNING FOR TEACHING- I

Structure

9.1 INTRODUCTION

A teacher makes use of a number of methods and techniques to bring about effective learning. The techniques include, motivating the students, explaining, questioning, writing on the blackboard, using teaching aids and so on. The teacher could also make use of nonverbal behaviours such as smiling, nodding and gesturing. These groups of activities are called skills. A teaching skill is a group of teaching acts / behaviours intended to facilitate pupil's learning directly or indirectly (Passi). If the student teachers are conscious and aware of teaching skills, they will be able to concentrate on each of these skills and gain mastery over the skill. This unit introduces Microteaching to the student teachers and allows the student teachers to practise each skill one at a time until he or she becomes proficient in the skill. Later on, the student teachers will be able to link many such skills to achieve the desirable outcome.

9.2 OBJECTIVES

At the end of the unit, the student-teachers will be able to

- Realise the need for Microteaching to the student teachers.

NOTES

- Define Microteaching.
- Understand the process of microteaching.
- Appreciate the characteristics of microteaching.
- Understand the microteaching skills with its components.
- Understand the techniques involved in the microteaching skills.

9.3 MICROTEACHING

9.3.1 Microteaching – Definitions

Microteaching has been defined in different ways:

Allen D.W. (1966) defined microteaching "as a scaled down teaching encounter in class size and class time".

Allen Eve (1968) defined microteaching "as a system of controlled practice, that makes it possible to concentrate on specific teaching behaviour and to practise teaching under controlled conditions".

Clift and others (1976) described microteaching "as a teacher training procedure which reduced the teaching situation to a simpler and more controlled encounter achieved by limiting the practice of teaching to a specific skill and reducing teaching time and class size".

Buch, M.E (1968) defined microteaching as a "teacher education technique, which allows teachers to apply clearly defined teaching skills to carefully prepared lessons in planned series of 5 to 10 minutes. It encounters with a small group of real students, often with an opportunity to observe the results on videotape."

Passi B.K. stresses that "the most important point in microteaching is that teaching is practised is terms of definable, observable, measurable and controllable teaching skills."

Allen and Ryan (1969) while defining microteaching specified the following essential propositions.

- Microteaching is real teaching but complexities of normal classroom are simplified
- There is emphasis on training for the accomplishment of specific tasks
- There is increased control of practice
- Normal knowledge of results or feedback dimension is greatly exposed.

From the above stated definitions a more comprehensive definition of microteaching can be stated as follows. Microteaching is a

teacher training technique where the complexities of the normal classroom teaching are reduced by:

- Practising one teaching skill at a time
- Limiting the content to a single concept
- Reducing the class size to 5 to 7 (real or peer) and
- Reducing the duration of the lesson to 5 to 7 minutes

9.3.2 Characteristics of Microteaching

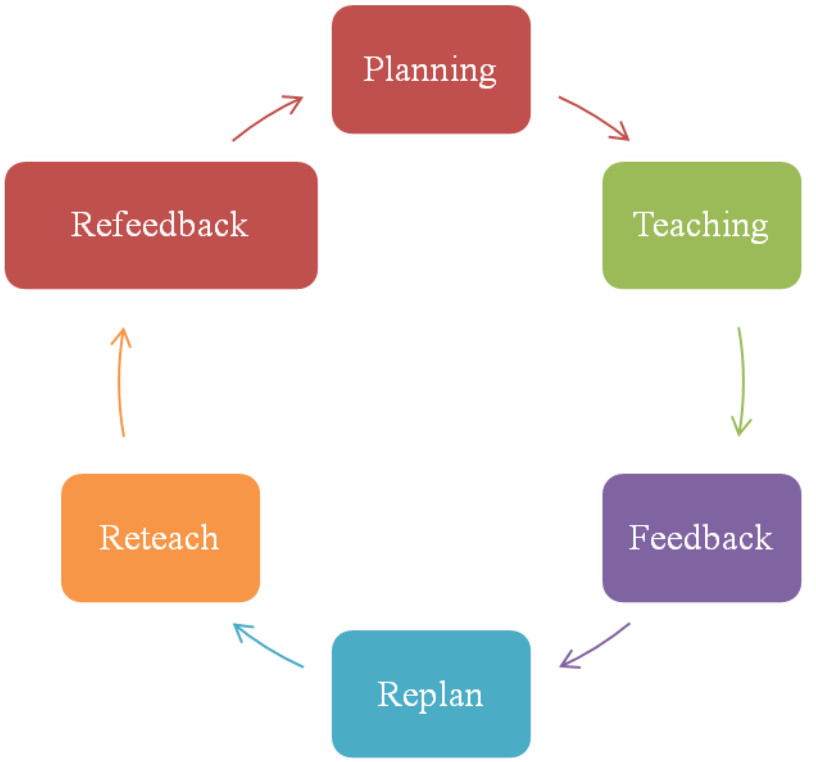
From the definitions stated above, the characteristics of microteaching can be summarised as follows:

- Microteaching is a teacher training technique and not a teaching method.
- Microteaching is a real teaching, though the teaching situation is simulated
- In microteaching the teacher trainee practises one specific teaching skill at a time, till he/she attains mastery over the skill
- Microteaching is a scaled down teaching encounter in class size (5-7 students), content (one concept) and class time (5-7 minutes).
- Microteaching operates on a predecided model: Plan, Teach, Feedback, Re-plan, Re-teach, Re-feedback, Re-plan etc.
- Microteaching allows for increased control of practice by providing feedback to the teacher-trainees.
- Microteaching is not a substitute, but a supplement to the teacher-training programme.
- Microteaching is a cyclic process. The predecided model (Plan, Teach, Feedback, Re-plan, Re-teach, Re-feedback) is repeated till the trainee achieves the expected level of mastery.

9.3.3 Process

The practice phase is the most important phase of the microteaching programme. In this phase the teacher trainee chooses a specific skill, prepares a micro-lesson plan and teaches a small group of students (peers or real pupils) for a duration of 5-7 minutes. The teacher educator and the peer group observers rate the lesson using an observation schedule or an appraisal guide. On the basis of the performance appraisal, immediate feedback is given to the teacher-trainee by the observers. The trainee then modifies her lesson and reteaches to another set of students. This lesson is also rated by the supervisor and other observes and then analysed and discussed with the trainee. This process is repeated till the

trainee attains adequate level of the skill acquisition. The completion of these steps results in the completion of one microteaching cycle as shown below



Steps in Microteaching Cycle

The steps in microteaching cycle can be listed as under.

i. Planning: This involves selection of the skill to be practised, awareness of the components of the skill, selection of a suitable concept and the writing of a micro lesson plan.

ii. Teaching: The trainee teaches the lesson in the microteaching setting. NCERT has suggested the following setting for microteaching.

Time : 6 Minutes

Number of students : 5 to 10; Real pupils or preferably peers

Supervisor : Teacher educator and/or one or two peers

The lesson is being observed by the teacher supervisor and/or peers or videotaped or audio taped.

iii. Feedback: The observers analyse the performance and discuss it with the teacher trainee on the basis of their ratings using the appraisal guide. The feedback should focus on specific behaviour related to the model of the teaching skill. The

supervisor can reinforce effective behaviour and draw to other behaviour modifications necessary for mastering the skill.

iv. Replan: In the light of the feedback received from the supervisor and peer observers the teacher trainee replans her micro-lesson by writing another micro-lesson plan or modifying the existing one.

v. Reteach: The teacher-trainee reteaches the revised lesson to another, but comparable group of students. The supervisor checks to see whether there is any improvement in skill attainment.

vi. Refeedback: The supervisor assesses the lesson once again and provides the feedback to the trainee. This process repeats till the teacher trainee acquires the required level of competency.

Check Your Progress

Notes : a) Write your answers in the space given below
b) Compare your answer with the one given at the end of the unit

1. Define Microteaching
   ……
   ……
2. State any two characteristics of Microteaching.
   ……
   ……
3. What are the steps in the Microteaching cycle.
   ……
   ……

9.4 MICROTEACHING SKILLS

9.4.1 SKILL OF EXPLANATION

A teacher is said to be explaining when he is describing 'how', 'why' and sometimes 'what' of a concept, phenomenon, event, action or condition. It can be described as an activity to bring about an understanding in the learner about a concept, principle etc. While explaining the teacher gives:

- Causes for some phenomena, event or action
- Reason for some phenomena, event or action
- Steps involved in arriving at a result or
- Various events that have resulted in phenomena

Explaining bridges the gap in understanding the new knowledge by relating it to the past experience. Thus explaining depends upon the type of the past experience, the type of the new knowledge (i.e. the concept, the principle, the phenomenon etc) and the type of the relationship between them.

Explanations can be made more effective by using simple and clear language for clarity, examples and illustration materials for better understanding and appropriate link words for relating the concepts.

The following behaviours are desirable for explanation:

- Gaining attention, establishing rapport and maintaining interest.
- Clearly stating the aim.
- Clearly stating the concepts.
- Developing the explanation from (i) known to unknown concepts/rules or

  (ii) examples to rules, or (iii) rule-example-rule.
- Using simple and suitable language.
- Maintaining the logical sequence and continuity.
- Modulating voice to convey the relative importance of the content.
- Proving occasional summaries to refocus attention.
- Using relevant examples and illustrations for clarifying major ideas.

Components of the Skill of Explanation

The main components of the skill are as follows

- Beginning statement
- Clarity
- Fluency
- Use of link words
- Planned repetition
- Stimulating questions
- Concluding statements

Explanation: components and Description

NOTES

Check Your Progress

Notes : a) Write your answers in the space given below
b) Compare your answer with the one given at the end of the unit

7. Define unit

……
……

8. State the characteristics of a good unit

……
……

9. What are the steps involved in writing a unit plan in mathematics?

……
……

10. List out the advantages of unit plan

……
……

Skill components	Description of Behaviour
Beginning statement	• Gaining attention, • Establishing rapport, • Arousing interest. • State the purpose of the explanation clearly by making statement such as, 'Today we are going to derive a formula' or 'prove a theorem', etc
Clarity	• State concepts clearly; • Define new terms and concepts; Use simple language; Maintain logical sequence and continuity of ideas; • Use suitable examples and illustration. • Develop the explanation from: (i) Known ⟶ Unknown (ii) Examples ⟶ Rule Or (iii) Rule ⟶ ⟶ Examples Rule

	• Voice modulation to emphasize important points.
Fluency	• Use appropriate vocabulary • Use simple language • Easy flow of ideas
Use of link words	• Use linking words and phrase to maintain continuity in the statements. Examples: therefore, similarly, that is why, since, because, in order to, hence, as a result of ... etc.
Planned Repetition	• Deliberately repeat some importance concepts or points • Provide occasional summaries to refocus attention
Stimulating Questions	• Check the understating by questioning • Stimulate and channelise the students thinking by thought. provoking questions, (Example: 'How'or 'Why' type)
Concluding Statement	• Consolidate the ideas by summarising. • Conclude the explanation by statements like 'Thus we have proved the Pythagoras theorem' 'This is how we construct a rhombus' etc

Undesirable behaviour

- **Making irrelevant statement**

  Statements which are not related to and do not contribution to the understanding of the concept being explained are irrelevant for explanation. Such statement distract the learners and lead to confusion and hence should be avoided.

- **Lake of continuity**

Lack of continuity in terms of logical sequence, relationship with previous statement, references to earlier experiences and so on has to be avoided.

Model micro lesson in skill of explaining

Name of the teacher-trainee	: Divya
Class	: Peer group
Skill	: Explanation
Lesson / topic	: Mensuration
Concept	:Derivation of the formula for the area of an equilateral triangle

Components:

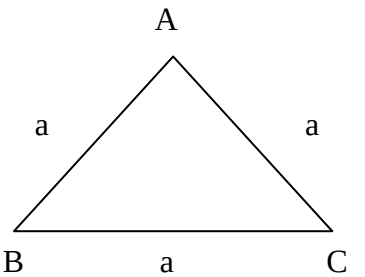
- Beginning statement
- Clarity
- Fluency
- Use of link words
- Planned repetition
- Stimulating questions
- Concluding statement

Teacher: In the previous class we have derived a formula to find the area of a right-angled triangle. What is that the formula for the area of a right angled triangle? (1)

Student : 1/2ab, a & b are the side containing the right angle

Teacher : Today we are going to derive a formula for the area of an equilateral triangle. (1)

The teacher draws an equilateral triangle ABC on the blackboard and marks the sides as ‘a’ units (2)



Teacher : What kind of a triangle is ABC? Why? (1)

Student : Triangle ABC is equilateral because all sides measure 'a' units

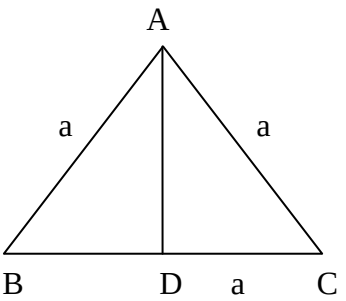
Teacher : How do you find the area of triangle ABC? (6)

Student : Using the formula ½ bh.

Teacher : What is the base and the altitude for triangle ABC? (6)

Students : BC is the base and the altitude has to be drawn.

The teacher draws AD perpendicular to BC (2)



Teacher : In ΔBC, BC is the base. AD is the drawn perpendicular to BC.

Therefore (4) AD is the altitude.

By (4) Applying the formula for the area of a $\triangle$, we have

Area of $\triangle$ABC = 1/2 BC × AD (3)

Since (4) BC = a, we have

Area of $\triangle$ABC = 1/2 a × AD

Now we have to find AD and we will see how to find AD. (2)

Teacher : What kind of triangles are ΔABC and ΔACD? Why? (6)

Student : ΔADB and ΔACD are right angled triangles because $\angle$ADB = $\angle$ADC = 90˚

Teacher : Good. Since (4) AD is drawn perpendicular to BC, $\angle$ADB = $\angle$ADC = 90˚.

Therefore (4) ΔADB and ΔACD are right-angled triangles (2)

Teacher : Now consider any one of the triangles, say ΔADB. In ΔADB, $\angle$ADB = 90˚.

Since (4) AB is opposite to the right angle $\angle$ADB, AB is the hypotenuse. Therefore by applying Pythagoras theorem, (4) we have $AB^2=BD^2+AD^2$ (3)

Teacher : How do you get $AB^2 = BD^2+AD^2$? (6)

Student : By Pythagoras theorem, square of the hypotenuse = sum of the squares of the other two sides.

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Teacher : Good. In ΔABD, AB is the hypotenuse, BD and AD are the sides containing the right angle. Hence we have

$AB^2=BD^2+AD^2$ (5)

Using the above relation we can find AD. By taking AD^2 to L.H.S of the equation we have, $AD^2 = AB^2 - BD^2$

By substituting for AB we have $AD^2 = a^2 - BD^2$ (3)

Teacher : How do you find BD? (6)

Student : Silence

Teacher : Now we will see how to find BD. The teacher uses cut out of an equilateral ΔABC and asks one of the students to fold it along AD. (2)

Teacher : As we fold the triangle ABC along AD, the point B falls exactly on C.

Therefore (4) D is the mid points of BC.

The teacher demonstrates and repeats it once again. (2) and (5)

Teacher : Since (4) D is the midpoint (4) BD = $\frac{BC}{2} = \frac{a}{2}$. Hence (4) we have BD = $\frac{a}{2}$

Teacher : Very good. Now we can find AD by substituting, BD = $\frac{a}{2}$, (5)

Thus we have,

$AD^2 = a^2 - BD^2$

$= a^2 - \left(\frac{a}{2}\right)^2 = a^2 - \frac{a^2}{4}$

By taking the L.C.M and simplifying we get (2)

$AD^2 = \frac{4a^2 - a^2}{4} = \frac{3a^2}{4}$

Since $AD^2 = \frac{3a^2}{4}$ by taking the square root we get AD.

Therefore AD = $\sqrt{\frac{3a^2}{4}} = \frac{\sqrt{3}a}{2}$ = (3)

Teacher : Repeat the steps once again (5)

Teacher : How did you get AD = $\frac{\sqrt{3}a}{2}$? (6)

Student : By applying Pythagoras theorem and substituting BD = $\frac{a}{2}$

Teacher : Having got AD = $\frac{\sqrt{3}a}{2}$ we can find area of ΔABC by substituting the value of AD.

Thus we have,

Area of ΔABC = 1/2 a × AD

$= 1/2a \times \frac{\sqrt{3}a}{2}$

$= \frac{\sqrt{3}\ a^2}{4}$ square units

Therefore Area of an equilateral triangle

$= \frac{\sqrt{3}\ a^2}{4}$, where 'a' is the side.

Teacher: What is the formula to find the area of an equilateral triangle? (6)

Student : A = $\frac{\sqrt{3}\ a^2}{4}$

Teacher : How many measurements do you need to compute the area of an equilateral triangle? (6)

Student : only one measure; the measurement of the side.

Teacher : Briefly summarises the derivation. (7)

Teacher : Thus we have derived the formula for the area of an equilateral triangle.

Area of an equilateral triangle = $\frac{\sqrt{3}\ a^2}{4}$ square units; where 'a' is the measurement of the side of equilateral triangle (7).

An Appraisal guide-Skill of Explanation

Skill of components	**Very good**	**Good**	**Average**	**Poor**	**Very poor**
Beginning statement					
Clarity					
Fluency					
Use of link words					
Planned repetition					

Stimulating question					
Concluding statement					

9.4.2 SKILL OF PROBING:

Probing is going deep into the pupil's responses by asking a number of questions about what they already know and to lead them to correct response or to remove any ambiguity or misconception, which has lead to such responses. Probing is to be done where there is no response, or increasing response, or partially right response.

Purpose of probing:

The specific purposes of probing are:

- to lead the pupils from a 'no' or 'wrong' response to the correct response by step- by-step questioning.
- to clarify pupil's understanding about the concept.
- to help the pupil to view the response from a broader perspective.
- to involve more and more pupils in the discussion, and
- to increase critical awareness in the pupils.

Techniques of probing:

Prompting: This technique is going deep into the pupil's initial response. This technique is employed when the student gives no response or an incorrect response. Prompting is leading the student from the initial response to the correct response with a series of hints or prompts through step-by –step questioning process.

Seeking further information technique: This technique involves leading a pupil from partially correct or incomplete response to the correct response through questioning.

Refocusing technique: This technique is employed when the student gives completely correct response. Refocusing through questioning helps the pupils to view the correct response from a different viewpoint.

Redirection technique: Redirection can be employed when a question has more than one correct answer or when there is no response, wrong response or partially right response. Redirection is posing a same question to many pupils to respond.

Increasing critical awareness technique: This technique involves putting questions such as 'how' and 'why' to increase the critical awareness of the pupils about the correct response. Thus this technique is followed when the students give correct response.

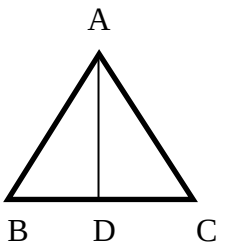
Components of the Skill of Probing

The main components of the skill are as follows

- Prompting
- Seeking further information
- Refocusing
- Redirecting
- Increasing critical awareness

Table gives the description of each components of the skill of probing questioning

Probing: Components and Description

Skill components	Description of behaviour
Prompting	• Giving hints or clues to lead the students from no response or wrong to correct response. Example: How do you prove that BD=CD in a triangle ABC?  Student: Silence Teacher: Do you think that you can apply congruency postulate to prove the equality of the sides?
Seeking further information	• Questions that lead the students from partially correct or incomplete responses to correct the answer. • Asking the questions to clarify, elaborate or explain his initial response. Examples: Can you put it in other words? Can you give me an example to

	support your view? Will you please elaborate your answer?
Refocusing	• Questions that help the student to view his correct response in a broader perspective • Questions that enable the pupil to relate his response with other similar situations • Questions that enable the pupil to consider the implications of a given response in a more complex and novel situations. Example: In what way are congruent triangles different from similar triangles? How do you relate a function and a relation? In what way is it similar to…..?
Redirection	Directing the same questions to the other pupils when there is a wrong response, incomplete response, partially right response, or while prompting or while seeking other information and so on.
Increasing critical awareness	• Asking 'How' and 'Why' of a correct response. • Questions seeking a relationale for the right response. Examples: Is the given relation a function? Why?

Undesirable Behaviour:

Avoid the following while asking questions:

- Frequent repetition of the questions and answer could make the students not attentive.
- Reframing the questions could result in confusion.

- Suggestive questions or 'echo' questions - questions based on the facts just stated do not require the students to think and can be avoided.

  Example: The teacher states the definition of a quadrilateral and poses the question. What is the name of the figure bounded by four line segments?

- Rhetorical questions: These are statements with interrogative phrases. The teacher does not expect an answer from the students.

  Example: Don't you think this is a complex problem? But this can be solved in different ways.

- Yes or No questions or Leading questions: Questions requiring the student to respond by 'yes' or 'no' are leading questions and do not stimulate them to think.

  Example: Is this a rectangle? Are these two factors equal? Is this an equilateral triangle?

Model Micro-lesson on the Skill of Probing Questions:

Components:

- Prompting
- Seeking further information
- Refocusing
- Redirection
- Increasing critical awareness

Name of the teacher trainee	: Amutha
Class	: Peer group
Skill	: Probing questions
Lesson/topic	: Mensuration
Concept	: Properties of a Rhombus

Teacher	: In your previous classes your have learnt about quadrilaterals. Name some quadrilaterals.
Ram	: Rectangle, Square, Parallelogram and Rhombus.
Teacher	: What is a rhombus?
Sita	: Quadrilateral
Teacher	: What are the properties of a rhombus? (2)
Ram	: All sides are equal.

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Teacher : Any other property? (2)

Ram : Angles are equal.

Teacher : Is Ram right, Rajesh? (4)

Rajesh : No

Teacher : Why? What is wrong? (5&1)

Rajesh : Silence.

Teacher : Is rhombus a parallelogram? (1)

Rajesh : Yes

Teacher : Why is it a parallelogram? (5)

Rajesh : Because in a rhombus opposite sides are parallel and equal.

Teacher : What are the properties of a parallelogram? (2)

Raghu : In a parallelogram opposite sides are parallel and equal.

Teacher : Good. How are the angles in a parallelogram? (4)

Priya : Opposite angles are equal

Teacher : How are the angles in a rhombus? (3)

Ram : Opposite angles are equal

Teacher : Why? (5)

Ram : Because rhombus is parallelogram

Teacher : Very good. How do you compare a rhombus with a square? (3)

Sudha : In both square and rhombus all sides are equal. In a square all angles are equal to 90°

Teacher : Good. What are the properties of a rhombus?

Priya : Rhombus is a parallelogram. All sides are equal, opposite angles are equal

Teacher : Good.

Appraisal Guide: Skill of Probing

Skill components	**Very poor**	**Poor**	**Average**	**Good**	**Very good**
Prompting					
Seeking further information					
Refocusing					

Redirection					
Increasing critical awareness					

9.4.3 SKILL OF STIMULUS VARIATION

It is very important for a teacher to secure and sustain pupil's attention. For this purpose the teacher uses some gestures, body movements, makes certain verbal statements etc. All these behaviours are related to stimulus variation. The skill of stimulus variation can be defined as deliberate change in the attention drawing behaviours of the teacher in order to secure and sustain pupil's attention towards the lesson.

Desirable Behaviours

- Movement of the teacher from one spot to another with a specific purpose.

  Example: Moves to the blackboard to write something on the board. Moves towards the back of the classroom to check what the students are doing. Moves towards a particular student to clarify some doubts.

- Gestures, body movements and facial expressions to direct attention, to emphasize importance, to explain emotions, to indicate size, shape etc.
- Voice modulation to pay special attention to a particular point.
- Focusing pupils' attention on a particular point or concept by using (i) verbal statement like 'look at the blackboard', 'see the figure in the chart' etc. (ii) gestures and (iii) both verbal statements and non-verbal cues.
- Change in interaction style from 1) teacher to group 2) teacher to student 3) student to student.
- Introducing deliberate silence of 3 to 4 seconds.
- Changing the sensory focus
- Physical involvement of the students

Example: Asking the student to work out a problem on the blackboard; seeking student's involvement in doing a demonstration; getting students help in holding a chart and so on.

Components of the skill

- Teacher movement
- Teacher gesture
- Change in sensory focus

- Change in speech pattern
- Verbal pupil pattern
- Physical pupil pattern

Stimulus Variation: Components and Description

Skill Components	Description of Behaviour
Teacher movement	• Purposeful movement of the teacher from one spot to another
Teacher gesture	• Nonverbal cues like body movements, facial expressions etc. to enhance the value of a message.
Change in sensory focus	• Focusing pupils attention • Making use of the different senses like hearing, seeing, touching and so on. • Changing from aural to visual, from visual to aural or a combination of aural and visual.
Change in speech pattern	• Modulating the voice • Variation in time, pitch, speed of voice to emphasis certain points. • Deliberate pause of 3 to 4 seconds.
Verbal pupil pattern	• Involvement of the students through verbal communication style as: (i) teacher to group (ii) teacher to student (iii) student to student • Example: Asking questions, discussion etc.
Physical pupil pattern	• Variation in physical involvement of the students Examples: Student moves towards the blackboard to work out a problem. Student holds the chart. Student helps the teacher in demonstrating an experiment.

Model Micro-Lesson: Skill of Stimulus Variation

Name of the teacher Trainee : Priya

Class : Peer group

Skill : Stimulus variations

Lesson/Topic : Menstruation

Concept : Area of a rhombus

Components

- Teacher movement
- Teacher gesture
- Change in sensory focus
- Change in speech pattern
- Verbal pupil pattern
- Physical pupil pattern

Teacher : Today we are going to derive a formula for the area of a rhombus. Teacher moves towards the blackboard and writes the topic on the blackboard.(1)

Teacher : Ram, come and draw the figure of a rhombus on the blackboard.

Ram : Draws the figure and names it ABCD (6)

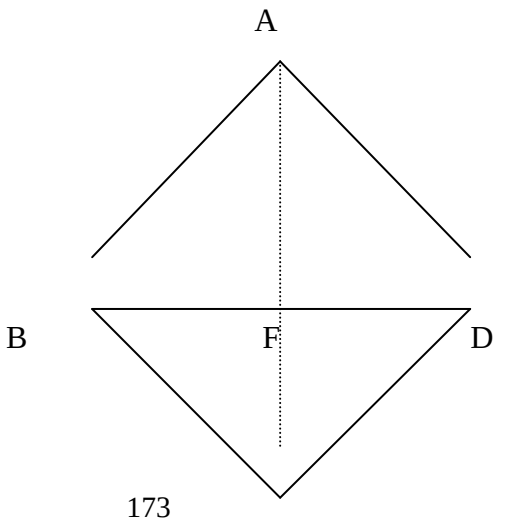
Teacher : Good (smiles) (2). Now look at the figure (3) and "tell me whether the rhombus can be divided into two triangles" (stresses triangles) (modulates the voice) (4)

Student : By drawing a diagonal we can divide the rhombus into two triangles. (5)

Teacher nods her head approvingly (2).

Teacher calls one of the students to draw the diagonal. (6)

Student moves to the blackboard and draws the diagonal (6)



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Teacher moves to the blackboard (1) and shows the two triangles (3)

Teacher : See the figure on the Blackboard (3). The area of the rhombus is equal to (stresses and modulate the voice) the sum of the areas of the two triangles) ABD and BCD. (4)

Teacher : The teacher points at the figure. (3) How do you find the area of Δ ABD?

Student : Area of ΔABD= ½ bh (5)

Good. (Teacher nods her head) (2)

Teacher moves towards the student (1)

Teacher : How do you find 'h'?

Student : Draw the altitude for the triangle (5)

Teacher moves towards the board (1) draws AF perpendicular to BD.

Teacher : Look at the figure (3) (pauses) (4). Which is the altitude? (Modulates the voice) (4)

Student : AF is the altitude (5)

Teacher : Similarly (stresses) (pause) (4) how do you get the altitude for ΔBCD

Student : Draw a perpendicular from C to BD (4)

Teacher calls one of the students to draw the altitude for the ΔBCD. Student moves towards the blackboard and draws CF perpendicular to BD (6)

Teacher : Thus (modulates the voice) (4) we have, Area of ABCD = ½ BD x AF + ½ BD x CF Teacher with hand movement (2) shows ½ BD is a common factor. Teacher calls one of the students to do the simplification

Student : Area of ABCD = ½ BD (AF + CF) (6)

Teacher : Look at the figure (3) and tell me what does BD represent?

Student : BD is a diagonal (5)

Teacher : What is AF + CF (stresses) (4) equal to?

The teacher moves to the blackboard (1) and moves her hand (2) over AF and CF to show AF + CF is another diagonal.

Teacher : Therefore (pause) (modulates the voice) (4) Area of rhombus = ½ d_1d_2

where d_1 and d_2 are the diagonals.

Appraisal Guide: Stimulus Variation

Skill Components	Very Poor	Poor	Average	Good	Very Good
1.Teacher movement					
2.Teacher gesture					
3.Change in sensory focus					
4.Change in speech pattern					
5.Verbal pupil pattern					
6.Physical pupil pattern					

Check Your Progress

Notes : a) Write your answers in the space given below

b) Compare your answer with the one given at the end of the unit

4. State the components of Skill of stimulus variation.

……

……

5. Mention any two purposes of skill of probing.

……

……

6. What are the main components of the skill of probing?

……

……

9.5 LET US SUM UP

Microteaching is an effective device for modifying the behavior of teachers under training. Microteaching is a teacher training technique and not a teaching method. In microteaching the teacher trainee practices one specific teaching skill at a time, till he/she attains mastery over the skill. This unit dealt the definitions, characteristics and process of microteaching. In this unit, the focus is on three Microteaching skills and its components.

9.6 UNIT-END ACTIVITIES

1) What is microteaching? Explain the characteristics of microteaching.
2) Draw microteaching cycle and explain it.
3) Explain the Skill of stimulus variation with its components.
4) What are the techniques involved in the skill of questioning.
5) Explain the components of the Skill of Explaining.

9.7 ANSWERS TO CHECK YOUR PROGRESS

1. Allen D.W. (1966) defined microteaching "as a scaled down teaching encounter in class size and class time".
2. The characteristics of microteaching:
   - Microteaching is a teacher training technique and not a teaching method.
   - Microteaching is a real teaching, though the teaching situation is simulated
3. Steps in Microteaching Cycle
   - Planning
   - Teaching
   - Feedback
   - Replan
   - Reteach
   - Refeedback
4. Components of the skill of stimulus variation
   - Teacher movement
   - Teacher gesture
   - Change in sensory focus
   - Change in speech pattern
   - Verbal pupil pattern
   - Physical pupil pattern

5. The specific purposes of probing are:

- to clarify pupil's understanding about the concept.
- to help the pupil to view the response from a broader perspective.

6. The main components of the skill of probing are as follows
    - Prompting
    - Seeking further information
    - Refocusing
    - Redirecting
    - Increasing critical awareness

9.8 SUGGESTED READINGS

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NOTES

UNIT –X
PLANNING FOR TEACHING- II

Structure

10.1 INTRODUCTION

Every student-teacher should develop skill in preparing a lesson plan property. The success or failure of a class largely depends upon how well or ill is the preparation on the part of the teacher. Proper planning will avoid waste of time, energy and unnecessary repetition. It is the core, heart of effective teaching.

Unit planning advocates the division of subject matter or content into small but meaningful units. With the help of this plan the teacher can arrange his material in sequential order and develop an effective teaching – learning process.

This unit covers the importance and preparation of lesson plan and unit plan. In this unit, you are exposed to the various components of a lesson plan in mathematics.

10.2 OBJECTIVES

At the end of the unit, the student-teachers will be able to

- Realise the need for writing lesson plans.
- Appreciate the role of planning for effective instruction in mathematics.
- Develop a unit plan.
- Prepare a lesson plan.
- Understand the components of a lesson plan.
- Understand the meaning and advantages of a unit plan.
- Understand the definition and significance of a lesson plan.

10.3 LESSON PLAN

The day-to-day planning of the details and sequence of each day's work is of great importance to each individual teacher. The daily lesson plan forces a teacher to determine what learning activities will go on in the class during that period. A teacher who goes to the class without planning for the lesson runs the risk of wasting time and dissipating effort. Indeed the very act of writing the plan out forces a crystallisation of the plan in the teacher's mind. This in itself is an important step towards a successful consummation of what being planned.

10.3.1 Definition of Lesson Plan

Good defined a lesson plan as an "outline of the important points of a lesson arranged in the order in which they are to be presented to students by teacher".

Davies defined lesson plan as "an arranged of learning, planning, organizing and controlling by a teacher". In the words of stands, Lester B "A lesson plan is actually a plan of action. It therefore includes the working philosophy of the teacher, her knowledge of philosophy, her information about and understanding of her pupils, her comprehension of the objectives of education , her knowledge of the material to be taught , and her ability to utilize effective methods" .

According to Bossing, "a lesson plan is the title given to a statement of the achievement to be realised and the specific means by which these are to be realised as a result of the activities engaged in during the period. It points out what has been done, in what direction the

pupils should next be guided and helped and what work is to be taken immediately. It is the teacher's mental and emotional visualisation of the classroom experience as he plans it to occur" it is in many ways the core, the heart of effective teaching.

10.3.2 Basic Considerations for Preparing the Lesson Plan

While preparing a lesson plan, the following board principles should be borne in mind.

- Selection of the suitable subject matter.
- Stating the objectives to be realised in clear and specific terms
- Assessing of the pupil's capacities, capabilities interests, background and previous knowledge
- Careful estimation and selection of those activities most appropriate for realising the objectives
- Arrangement of these activities into a properly ordered schedule
- Evaluating the attainment off the state objectives

10.3.3 Functions of a Good Lesson Plan in Mathematics

A good lesson plan in mathematics serves the following:

- Lesson planning makes the teacher's work more regular, organised and systematic
- It delimits the teacher's field of work and thus enables him to define his aims and objectives more clearly.
- It forces consideration of goals and objectives, the selection of subject matter, the selection of procedure, the planning of activities and the planning of evaluation devices.
- It prevents waste of time, as every step has been planned with forethought. Unnecessary repetition is thus avoided.
- It enhances self-confidence of teacher as it paves the way for the teacher to enter the class without anxiety.
- It organises and systematizes the learning process. As the hierarchy of lesson becomes well-knit, inter-connected and associated, the learning process takes a definite shape and its continuity is assured.
- It provides opportunities for the manifestation of creative and constructive urges.

- It facilitates appropriate use of teaching aids at appropriate places.
- It helps to develop reasoning, thinking and imagination power of the students.
- It helps sustain students' interest in learning by providing interesting and relevant learning experiences suitable to the students' level.
- Lesson planning helps in budgeting of time according to the requirement of children and syllabus.
- It helps to pick and choose the particular aspects that need emphasis.
- Lesson planning helps in establishing proper correlation between the different branches of mathematics.
- It helps in providing drill and practices in mathematical principles and formulae
- It keeps the teacher on the right track as she is conscious of every step that she has to take.
- It helps the teachers in selecting and using more relevant and appropriate, illustrative material to make the abstract mathematical concepts concrete.
- Planning helps the teacher to visualise students' difficulty and plan for remedial programmes.

10.3.4 Important Feature of a Good Lesson Plan

A good lesson plan should have certain important characteristics.

- All the objects of the lesson, both general and specifics, should be statedclearly in the lesson plan.
- A good lesson plan should outline in detail the various steps that the teacher proposes to take in the class.
- A good lesson plan should not remain at the oral or mental stage; it should be preferably written.
- A good lesson plan should have its basis on the previous knowledge and experiences of the learner and the present knowledge should be well integrated with the previous knowledge.
- Ample provisions should be made in the lesson plan for arousing the curiosity and sustaining the interests of the students.

- The subject matter presented, the materials used, and the teaching aids selected should cater to the individual needs and abilities of the students.
- The lesson plan should reveal the development of topics.
- Good lesson planning requires that the lesson unit must be finished within the time allocated.
- A good lesson plan should contain the questions to be asked, illustrations to be used, the assignments to be done and the activities to be carried out.
- The teaching techniques to be used by the teacher should be cleared explained in the lesson plan.
- A good lesson plan should indicated the list of A.V aids to be used and should be specify when and how to use them.
- Application stage for applying the learned rules and formulae should find a place in the lesson.
- A good lesson plan should stimulate reflective thinking, independent thinking and originality of expression on the part of the students.
- A good lesson plan should be flexible, and not rigid. It should make the presentation more effective
- A lesson plan should provide for consolidation and recapitulation of ideas.
- References should be clearly given.
- It should suggest activities to meet individual differences.
- It should provide intrinsic and extrinsic motivation.
- A good lesson plan should provide the basis for further learning.

Check Your Progress

Notes : a) Write your answers in the space given below
b) Compare your answer with the one given at the end of the unit

1. Define lesson plan.
   ……
2. List out the functions of a lesson plan.
   ……
3. State any five important features of a good lesson plan
   ……

10.3.5 Various Approaches to Lesson Planning

A teacher can follow any of the following approaches for lesson planning.

- HerbatianApproache
- Morrison's or Unit Approache
- Bloom's or Evaluation Approache
- RCEM Approache

Herbatian Approach

John Fredrik Herbat, a German philosopher and educationist (1776-1841) advocated pedagogy-based lesson planning. Herbatian approach to lesson planning involves the following steps.

- Preparation
- Presentation
- Association or Comparison
- Generalisation and
- Application.

Preparation

It pertains to preparing and motivating the children to learn the new topic. The mind of the child should be prepared to receive new knowledge. The preparation should not only set the atmosphere for learning, but it should also arrest attention of the students and focus it on the new topic.

The preparation may involve:

- Testing the previous knowledge relevant for learning the new topic
- Integrating the previous knowledge with the new lesson to be learned.
- Capturing the attention and maintaining interest using A. V. aids, story telling, etc. Arousing the curiosity of the students by creating a problematic situation or posing an intriguing question.
- Announcing the aim of the lesson. (For example, a statement like "Today we are going to derive a formula for the area of an equilateral triangle.")

Presentation

It is at this step that the actual teaching takes place. The students acquire new knowledge and ideas. For an effective learning outcome, the teacher should ensure active student participation by providing a number

of learning activities. The teacher can make use of audio-visual aids and illustrative materials to make the learning interesting, effective and meaningful. The teacher should stimulate the mental faculties of the students by asking thought- provoking questions. This is very important for teaching mathematics. The teacher can gradually build the new concepts based on the responses elicited from the students.

Association or Comparison

The new knowledge becomes more meaningful when it is compared, contrasted, associated and integrated with already existing knowledge. This step is particularly significant in subjects like mathematics where the students have to learn definitions, establish principles or generalisations or formulae deducted from already learnt concepts, postulates, theorems, and axioms. Some examples can be given and students may be asked to observe and compare and see how these are explicitly related to previously learnt ideas and concepts.

Generalisation

In mathematics the students are often required to establish some formula, law, generalisation or the rule. This is possible by presenting particular examples and requiring the students to observe and compare for common elements or pattern. Thus they are led to frame a general law or principle. The teacher's job is to enable the pupils to draw out the generalisation. When the mind comprehends new knowledge, it compares and contrasts with what is already present and only then ideas are generalised and new rules/laws or formulae are formed. This step enables the learner to systematise the new knowledge acquired.

Application

Knowledge is power only when it is applied to new and unfamiliar situations. It is particularly so in mathematics as the students have to apply the rules, formulae or generalisations that they have learnt in order to solve problems. By application the validity of the generalisation is tested and verified. Thus the knowledge becomes more meaningful to the learner and becomes permanent in his mind.

Recapitulation

Though recapitulation is not mentioned as a separate step in Herbartian scheme, the significance of recapitulation cannot be overlooked. This is used for ascertaining how well the students have understood the concepts and to assess how effective the method of teaching has been. Recapitulation can be done by.

i) asking questions on the contents of the lesson.
ii) giving a short objective type test.
iii) asking the students to arrange the steps in solving problem in its logical sequence, etc.

Recapitulation also helps the students to review and reorganise the subject matter and consolidate ideas.

Unit Approach (Morrison's Approach)

The Unit Approach of lesson planning is propounded by Prof. Morrison (1871-1945). This approach emphasises the mastery over the subject matter by the students. The subject matter or the lesson chapter is split into meaningful small segments known as units. The unit approach of lesson planning involves the following steps.

i) Exploration
ii) Presentation
iii) Assimilation
iv) Organisation
v) Recitation

Exploration

The teacher explores various methods and possibilities to motivate the students, to arouse the curiosity and to maintain the interest of the students. This the preparation step where the teacher could plan for the success of the lesson.

Presentation

This step is basically common to all the approaches of lesson planning which involves the selection and use of different methods to present the subject matter to the students.

Assimilation

The third step involves intensive learning, and deep understanding of the subject matter that facilitate effective concretization of the concepts.

Organisation

This is an important step in unit approach to lesson planning as it determines the extent to which students are able to reproduce the material of the unit in writing without any external help. The ability of the teacher to enable his students to reproduce the knowledge reflects the efficiency of the teacher.

Recitation

Recitation in unit lesson planning means that an individual student is able to reproduce the same text orally on the completion of the lesson by a teacher.

Bloom's Evaluation-based Approach to Lesson Planning

Bloom's approach is based upon the interrelationship among objectives, learning experiences and evaluation techniques. The approach involves the following steps.

- Formulating educational objectives
- Creating learning experiences
- Evaluating the changes of behaviour

Formulating Objectives

In this approach to lesson planning the first step is to formulate objectives and state them in clear and specific behavioural terms. This requires the teacher to assess the entry behaviour of the students so that he / she will be able to lead them towards the terminal behaviours which are otherwise the learning outcomes.

Creating Learning Experiences

After selecting and stating the specific outcomes of learning, the teacher has to plan and provide the most appropriate learning experiences that would result in the desirable changes in behaviour. This is the step where the teaching and learning takes place. These learning experiences-may be varied and different depending on the nature of the content, the learner and the type of learning outcome. This type of lesson planning forces the teacher to consider a variety of activities that are meaningful and goal-oriented. Such activities ensure greater student involvement in learning at every step and keep the students active and alert.

Evaluating the Changes of Behaviour

At this step the teacher selects suitable evaluation tools and techniques to assess the changes in behaviour. Thus evaluation becomes an integral part of teaching. As and when a learning experience is provided, its effectiveness in attaining the objective is immediately evaluated. If the outcome is not favourable, the teacher can plan an alternative learning experience to achieve the objective.

The format of a lesson based on this approach can be as follows.

Objectives	Learning Experiences	Evaluations

However, content is not given as a separate part, and other steps are not specified. But learning experiences are always based on the content and other steps can be easily incorporated.

RCEM Approach to Lesson Plan

RECM approach is advocated by the Regional College of Education, Mysore and hence the name. The rationale behind this approach is a system approach in education. A system approach demands the presentation of information in a systematic manner.

There are three stages or aspects in RCEM approach - They are:

- The input, otherwise known as Expected Behaviour Outcomes (EBOS)
- Process, otherwise known as communication strategy and
- Output, otherwise known as Real Learning Outcomes (RLOS) Input (EBOS)

The first stage in RCEM approach is concerned with the identification of the behavioural objectives pertaining to a particular lesson or content

Process

This aspect of the lesson planning involves presentation and integration of knowledge and skills. The emphasis is on how well the knowledge and skills are communicated to the learners

The Output or RLOS

The output indicates the terminal behaviour or change in behaviour of the students after learning. The output stage in lesson planning represents the evaluative phase in the lesson.

Prerequisites for a Good Lesson Plan in Mathematics

However, all the approaches discussed above are tentative guidelines and need not be followed rigidly. All the above approaches emphasise the following as prerequisites for any lesson plan.

- Clearly and precisely stated objectives
- An interesting motivation to introduce the lesson
- Systematic and orderly presentation of the subject matter
- A suitable method with relevant learning activities, adequate teaching aids, examples etc
- Application of the new knowledge to new and daily life situations and integrating it with previous knowledge
- Reviewing and consolidating the learning points
- Evaluating the attainment of the objectives using appropriate evaluation devices.

10.3.6 Steps in Writing a Lesson Plan in Mathematics

For lesson plans in mathematics the following steps can be followed

- Introduction or Motivation
- Announcement of Aim
- Presentation
- Application
- Supervised Study
- Recapitulation and
- Assignment

10.3.7 Format of a lesson plan

Name of the trainee :

Subject :

Unit :

Topic :

Name of the school :

Standard :

Time :

Date :

General Objectives :

Specific Objectives :

Previous knowledge :

Aids Used :

References :

Steps / Content	Specifications	Learning Experience	Evaluation
Motivation Announcement of the Aim Presentation Application Supervised Study Recapitulation Assignment			

Check Your Progress

Notes : a) Write your answers in the space given below
b) Compare your answer with the one given at the end of the unit

4. What are the steps involved in writing a lesson plan in mathematics?

.....

10.4 Unit Plan

A Varity of meanings have been assigned to the term unit. Some of them are (i) the lesson for the day (ii) it is a block of work, (iii) as a chapter in a textbook, a project and (iv) as a method of instruction rather than a method of organisation of instructional materials, etc.

Writers of textbook and curriculum makers have always found it convenient and helpful to organize instructional materials in unit blocks or chapters. Some of them consider units as a collection of theorems and exercises to be learned within a specified period of time .Some other speaks of the units as a "series of exercises which are based upon certain postulated facts and which lend themselves to logical demonstrations to cover in time, the equivalent of six to eight weeks".

Therefore, a unit is a large segment of subject matter having a common theme or idea. A unit can be split up into smaller sub-units called topics and the topics are linked to one another by a common ideas or a principle. For example the unit 'circle' contains many topics such as

i. Circle-definition and elements

ii. Area and circumference of a circle

iii. Angle properties in a circle

iv. Tangents to a circle etc.

All these topics, are interrelated and interlinked by the common ideas, the 'concept of the circle'. The syllabus contains many units such as polynomials. Triangles, Equations, Business, Arithmetic, Mensuration, etc, to list a few.

10.4.1 Definition of a Unit

Preston defines a unit as "a large block of related subject matter as can be overviewed by a leaner".

According to Samford, "a unit is an outline of carefully selected subject matter, which has been isolated because of its relationships to pupil's needs and interests".

A unit is not just a block of subject matter, but it is a large subdivision of well organised subject matter. Units of instruction break

up a course into meaningful segments, that are large than lesson plans. They are organised around specific topics, which are neither blocks of subject matter nor a series of independent lesson. A unit represents a careful organisation of subject matter and learning experiences around a unifying principles of property.

10.4.2 Characteristics of a Good Unit

- A unit should be meaningful segment of well organised subject matter.
- A unit can be broken up into interrelated sub-units or topics.
- The segment in the unit should be linked together by a unifying idea or principles or property.
- A unit should not be too a lengthy or too short.
- The length of the unit should be such as to retain the interest of student.
- A good unit should be part of a unit that permits growth from year to year.

10.4.3 Steps in Unit planning

Unit planning is a part of year planning and it is a middle ground between course planning and lesson planning .It is longer than lesson planning, but shorter than course planning. Unit planning involves the following stages.

1. Content Analysis

After choosing the unit, the teacher has to do a detailed analysis of the contents oftheunit to get an in-depth knowledge of the terms, concepts, principles, generalisations constituting the unit .This helps the teacher to break up the unit into meaningful sub-units and lessons retaining the continuity throughout the unit.

2. Stating the general and specific objectives

The teacher should identify the general objectives and state the specific objectives or learning outcomes to be achieved as a result of learning the unit.

3. Planning the learning activities

The third step is to select suitable learning experiences that may lead to the realisation of the stated objectives keeping in mind individual differences, the psychology of learning, the content and objectives. Suitable learning activities can be planned, to which the students will be exposed to during the instruction of the unit .The teacher also has to plan specific teaching strategies that will be employed for each segment (sub unit) of the unit.

4. Evaluation procedure

The last step of unit planning is to select appropriate evaluation tools and techniques to assess the content coverage, the realisation of the stated objectives and the effectiveness of teaching strategies

10.4.4 Advantages of Unit Plan

Unit planning contributes to the educational process in the following ways.

- Unit plan beaks up a lengthy unit into smaller sub-units or topics so that pupils can easily grasp the scope of these during a brief overview.
- It helps the teacher to present the various principles and concepts constituting the unit in an orderly and systematic manner, without losing their continuity.
- It enables the pupils to see clearly the relationship between the various facts, processes and principles that make up the unit.
- It helps the pupils to appreciate the unifying principles linking all the information together in the unit. This guides pupils to view the sub-units as part of a whole and not independent segments of information.
- It helps the teacher to plan a variety of learning experiences, keeping in mind the individual differences, the nature of content and objectives to be achieved.
- It provides frequent opportunities for the students to review and reorganise their learning.
- It helps the teacher to plan definite outcomes of learning so that they are clear not only to the teacher, but also to the students.
- The study outline of the unit plan provides the students with directions as to what to study, and how to do it most effectively.

Format of Unit Plan

1. Subject -
2. Class -
3. Name of Unit -
4. Total Periods -
5. Duration of Period -
6. Content -
7. Number of sub-units -
8. Aids -

Each sub-unit has following steps:

Sub unit and Its Introduction	Teaching points	Objective and Expected behavior outcomes	Teaching Learning Situation			
			Teacher's Activity	Students Activity	Teaching Aids	Home Work

Sample Unit Plan:

1. Subject : Modern Mathematics
2. Class : 9^{th}
3. Name of Unit : Set Theory
4. Total Periods : 11 Periods
5. Duration of Period : 40 minutes
6. Content : Sets, types, set operations, solution oF problems.
7. Number of Sub Units : 6

S.No	Sub-Units	Number of Periods
1.	Introduction of set theory	1
2.	Methods of representing sets	1
3.	Types of set	3
4.	Set Operations	2
5.	Venn Diagrams	2
6.	Solution of Problems related to set theory	2
		Total 11

Subunits	Teaching Points	Objectives and expected behavior outcomes	Teaching Learning Situation			H.W	Evol
			Tr's Activity	Stds. Activity	Teaching Aids		

NOTES

8. Aids

Introduction of SetTheory	Meaning and Definition of sets	To gives the knowledge of Modern mathematics	P.K. Testing by teacher eg after Showing chart of tea set etc. The he announces the lesson	Students actively Participate incounting things from the chart	Chart showing Tea set, sofa set etc.	To find example of sets from daily life	1. What do you understand by set? 2. Give examples of sets	

Sub Unit – II

Describing set	Two types of representing a set.	To give the knowledge of how to represent a set.	P.K. Testing Announcement of the lesson. Students and teachers joint activity.	Students Note on their books.	Chart showing the two ways representing a set	A few questions on sets, to be represented in two ways.	How will you represent set of even nos. in set builder from? Represent set of odd nos. In tabular form.

Sub Unit-III

Types of sets.	Various types of sets like infinite, empty, sub set.	To give the knowledge of various types of sets and their notation.	P.K. Testing Teacher develops his lesson as usual.	Students listen carefully and note on their copies.	Flash cords showing various types of sets.	Questions on various types of sets.	1. What do you mean by finite set? 2. What do you mean by empty set?

Sub Unit –IV

Set operations	Union and Intersection of sets	To give the knowledge of various set operations.	P.K. Testing teacher develops his lesson as usual as it is done under lesson planning	Students take active participation in developing the lesson.	Chart showing different kinds of set operations.	Questions to find the intersection and union of sets.	1. Find Union of A={1,2,3,5} B={1,2,4,6} 2. Find Intersection of A={a,b,c} B={b,d,e}

Sub Unit- V

Venn Diagram	To explain the content of Venn	To give the knowledge of venn diagrams	P.K. Testing to give examples of various sets	Students help in developing the lesson	Charts and flash cards showing venn diagrams	Questions from the exercise related to venn diagram	1.Represent AIB through venn diagram 2.Represent A through venn diagrams.
Problems related to set theory	Solution to various problems related to set theory	To help the students to solve various problems	P.K. Testing to solve problems	Students take active participation in solving problems	Flash cards showing different problems	A few problems from the book.	A few problems related to the topic.

Check Your Progress

Notes : a) Write your answers in the space given below

b) Compare your answer with the one given at the end of the unit

6. Define unit

……

……

7. State the characteristics of a good unit

……

……

8. What are the steps involved in writing a unit plan in mathematics?

……

……

10.5 LET US SUM UP

In this unit, the focus is on planning for instruction. The daily lesson plan forces a teacher to determine what learning activities will go on in the class during that period. A teacher who goes to the class without planning for the lesson runs the risk of wasting time and dissipating effort. Indeed the very act of writing the plan out forces a crystallization of the plan in the teacher's mind. Basic considerations for preparing the lesson plan, functions of a good lesson plan in mathematics, important features of a good lesson plan, various approaches to lesson planning and steps in writing a lesson plan have been dealt in detail.

Unit planning is a part of year planning and it is a middle ground between course planning and lesson planning .It is longer than lesson planning, but shorter than course planning. Unit planning involves the following steps - Content Analysis, Stating the general and specific

objectives, Planning the learning activities and Evaluation procedure. The essential aspect of unit and lesson planning has been elaborated so that unit and lesson planning can be done effectively.

10.6 UNIT-END ACTIVITIES

6) What is a lesson plan? Explain the components of a lesson plan.

7) What is a unit plan? Discuss how it can be properly carried in the teaching of the subject mathematics.

8) Prepare a lesson plan for any one topic from secondary and higher secondary mathematics.

9) What are the advantages of unit plan?

10) Differentiate between unit plan and lesson plan?

10.7 ANSWERS TO CHECK YOUR PROGRESS

1. Good defined a lesson plan as an "outline of the important points of a lesson arranged in the order in which they are to be presented to students by teacher".
2. Functions of a Good Lesson Plan in Mathematics
   - Lesson planning makes the teacher's work more regular, organised and systematic
   - It delimits the teacher's field of work and thus enables him to define his aims and objectives more clearly.
   - It forces consideration of goals and objectives, the selection of subject matter, the selection of procedure, the planning of activities and the planning of evaluation devices.

3. Important Features of a Good Lesson Plan
   - All the objects of the lesson, both general and specifics, should be stated clearly in the lesson plan.
   - A good lesson plan should outline in detail the various steps that the teacher proposes to take in the class.
   - A good lesson plan should not remain at the oral or mental stage; it should be preferably written.
   - A good lesson plan should have its basis on the previous knowledge and experiences of the learner and the present knowledge should be well integrated with the previous knowledge.
   - Ample provisions should be made in the lesson plan for arousing the curiosity and sustaining the interests of the students.

4. Steps in Writing a Lesson Plan in Mathematics
   - Introduction or Motivation

- Announcement of Aim
- Presentation
- Application
- Supervised Study
- Recapitulation and
- Assignment

5. Various Approaches to Lesson Planning

- Herbatian Approaches
- Morrison's or Unit Approaches
- Bloom's or Evaluation Approaches
- RCEM Approaches

6. According to Samford, "a unit is an outline of carefully selected subject matter, which has been isolated because of its relationships to pupil's needs and interests".

7. Characteristics of a Good Unit

- A unit should be meaningful segment of well organised subject matter.
- A unit can be broken up into interrelated sub-units or topics.
- The segment in the unit should be linked together by a unifying idea or principles or property.
- A unit should not be too a lengthy or too short.
- The length of the unit should be such as to retain the interest of student.
- A good unit should be part of a unit that permits growth from year to year.

8. Steps in Unit planning

- Content Analysis
- Stating the general and specific objectives
- Planning the learning activities
- Evaluation procedure

10.8 SUGGESTED READINGS

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UNIT – XI
ASSESSMENT AND EVALUATION -I

Structure

11.1 INTRODUCTION

Evaluation is an act or process that allows one to make a judgment about the desirability or value of a measure. Evaluation is integrated with the whole task of education and its purpose is to improve instruction and not merely to measure its achievements. In its highest form evaluation brings out the factors that are inherent in student's growth such as proper attitudes and habits, manipulative skills, appreciations and understandings in addition to the conventional acquisition of knowledge.

A teacher has to devise and administer a variety of tools and techniques for evaluation. Achievement tests constitute an important tool of evaluation.

It is necessary for the teacher to know how far the pupils have attained in particular subjects.

This unit provides various forms of achievement tests that are used to measure the accomplishment of the pupil's and discusses the role of evaluation in the teaching and learning of mathematics and also presents various types of evaluation.

11.2 OBJECTIVES

At the end of the unit, the student-teachers will be able to

- Know the meaning of evaluation.
- Understand the purpose of assessment and evaluation.
- Describe different techniques of evaluation in mathematics.
- Recognize criterion referenced and norm referenced tests.
- Illustrate the use of evaluation in improving mathematics instruction.
- Illustrate various types of questions used for evaluation.
- Discuss the importance of blue print.

11.3 ASSESSMENT

By assessment, we mean the processes and instruments that are designed to measure the learner's achievement, when learners are engaged in an instructional programme of one sort or another. It is concerned with ascertaining the extent to which the objectives of the programme have been met. The term assessment is often used interchangeably with the terms evaluation and measurement. However, assessment has a narrower meaning than evaluation but a broader meaning than measurement. In its derivation, the word assess means "to sit beside" or "to assist the judge". It, therefore, seems appropriate in evaluation studies to limit the term assessment to the process of gathering the data and fashioning them into an interpretable form; judgement can then be made on the basis of this assessment. Let us take an example of testing of school children by Secondary Board. Tests are administered in reading, writing, science and other academic areas. Based on the information provided by the Secondary Board, educators, citizens and political leaders then make judgements about the effectiveness of the education system. Assessment, as we define it, precedes the final decision making stage in evaluation e.g., the decision to continue, modify, or terminate an educational programme.

11.4 EVALUATION

Evaluation in general is an act or a process that allows one to make a judgment about the desirability or value of a measure. Evaluation in educational situation is thus a relatively new term introduced to designate a more comprehensive concept of measurement

than is implied in conventional tests and examinations, the emphasis in evaluation being upon broad personality changes and more objectives of an educational programme and therefore include not only subject matter achievements, nit also attitudes, ideals, ways of thinking, work habits and personal and social adaptability. Thus evaluation is not just a testing programme. Tests are but one of the many different techniques that may contribute to the total evaluation programme. Evaluation is any systematic continuous process of determining.

- ❖ The extent to which specified educational objectives previously identified and defined are attained
- ❖ The effectiveness of the learning experiences provided in the class room
- ❖ How well the goals of education have been accomplished

Thus evaluation is integrated with the whole task of education and its purpose is to improve instruction and not merely to measure its achievement. In its highest form evaluation brings out the factors that are inherent in student's growth such as proper attitudes, habits, manipulative skills, appreciations and understanding in addition to the conventional acquisition of knowledge.

Evaluation is the process of finding out the extent to which the desired changes in behavior have taken place in the student. It differs from the concept of measurement in the sense that evaluation is more comprehensive. Measurement consists of rules for assigning number to attributes or characteristics of behavior whereas the evaluation aims at providing detailed and comprehensive meaning and interpretation to the behavioral attributes of a learner. It expresses quantitative as well as qualitative description of learner's performance. The purpose of evaluation is different at various stages of instruction. Prior to beginning of instruction, the assessment of the learners' present achievement should serve the basis for selecting and formulating instructional objectives and then for planning appropriate learning experiences. The evaluation also helps the teacher to know how effective the instruction has been in helping learners to master the "instructional objectives"

Effective evaluation of student's achievement with respect to accepted and planned objectives of instruction is considered an indispensable aspect of good teaching. Teachers use various evaluation procedures i.e., tests (oral and written), practical, assignments, observation, interview etc., for assessing and monitoring the progress of the students achievement in scholastic and co-scholastic areas. These evaluation procedures and techniques have become an integral part of the instructional process and influence students in many ways. One of the functions that evaluation serves is to enable students to determine how well they are learning and achieving. When students are aware of the

learning progress, their performances will be superior to what it would have been without such knowledge.

11. 4.1 Purpose of Evaluation

- ❖ To provide information for grading, reporting to parents and promoting students
- ❖ To evaluate the effectiveness of a single teaching method or to appraise the relative worth of several methods.
- ❖ To motivate the students.
- ❖ To select students.
- ❖ To evaluate the entire educational institution and to show how several of its aspects could be improved.
- ❖ To collect information for effective educational and vocational counseling.

11.4.2 Needs and Objectives of Evaluation

1. To help in laying out or bringing modification in the instructional and educational objectives of teaching mathematics for a particular grade in a specified teaching-learning situation.
2. To help in the selection and organisation of learning experiences for the teaching of mathematics related to a particular grade and teaching-learning situation.
3. To help in the selection and use of general and specific teaching-learning strategies and methods for the teaching of specific topics and branches of mathematics in a particular grade and teaching-learning situation.
4. To help in diagnosing the learning difficulties of the students (general and specific) and accordingly plan suitable remedial teaching.
5. To help the mathematics teacher for better planning and organising his teaching task according to the needs of his students and prevailing teaching-learning situations.
6. To help the teachers and students for getting due incentives and motivation (extrinsic as well as intrinsic) for carrying out their teaching-learning functions.
7. To help the guidance and counselling personnels for providing educational, personal and vocational guidance and counselling to the needy students.

8. To help in the process of realization of the teaching-learning objectives of teaching mathematics to a particular grade in a given teaching-learning situation.
9. To help the teachers, students, parents, guidance personnel, administrators, curriculum framers, researchers and planners to take needed decisions in their respective fields of work in the light of the results of evaluation.

11.5 CRITERION-REFERENCED AND NORM-REFERENCED EVALUATION

Evaluation task may also be categorized as criterion-referenced and norm-referenced.

Incriterion referenced evaluation, the tools of evaluation, tests and techniques are so constructed as to measure student learning according to a predetermined standard of achievement or performance. In other words, evaluation measure or a test based on reference to a "criterion" *i.e.* specific teaching-learning objective fixed in advance may be referred to as a criterion referenced evaluation. Such type of evaluation is deliberately conducted to provide results that can be directly interpreted in terms of the acceptable level of the performance *i.e.* previously specified level of performance. In no case it tries to compare a learner's performance with that of another or a group of learners as happens in the case of norm referenced evaluation or summative evaluation (discussed earlier). Its primary and sole purpose is to determine whether a learner has or has not acquired mastery over the specific learning task regardless of the fact that how other learners performed on the same task. In criterion referenced tests or evaluation measures, thus a criterion, an accepted level of performance (mastery level) is fixed well before the beginning of the actual teaching-learning session. In the end or anywhere during the instructional period, an attempt is made to know the extent to which predetermined level of performance in mastery level has been achieved by the individual learner. The results of such evaluation is thus interpreted in terms of the predetermined standard of absolute performance like. "Given ten, two digit additional problems the learner solves correctly atleast 9 within a period of 15 minutes."

The norm-referenced evaluation, as the name suggests, is based on reference to the 'norm' (i.e. in accordance with the norms for the results of all the learners who have undergone the same evaluation measure). That is why, such type of evaluation focuses on comparing the scores or performance of a learner with that of another or group of learners. It is in fact a competitive evaluation, which provides information relating to a student performance in comparison with other fellow students regardless of the fact whether he has acquired or not

acquired mastery or attained any specific level of performance over a learning task. The results of such evaluation are often used in declaring the students first, fifth or fifteenth in a particular class or assigning some grade level or showing that his scores or level of performance falls below or above in comparison to a regionally or nationally representive group's norm. Entrance test used for selection to some particular course or profession or promotion to higher post generally make use of such evaluation based on reference to the norm only because of their distinctive characteristic of making valid discrimination between the performances of the competitive individuals.

Difference between criterion-referenced evaluation and norm-referenced evaluation

Criterion-referenced and norm-referenced evaluation can be mainly distinguished on the basis of the method of interpretation employed for highlighting pupil performance. In criterion-referenced evaluation the basis employed for interpretation is to compare the individual's performance with a set standard or criteria of performance irrespective of one's position in the group. For example if in the construction of a mathematics unit test the criteria of absolute performance is laid down as "correctly solves 9 three digits additional problems out of the given 10 in 15 minutes" then the performances of all the students in the class will be evaluated on the basis of such laid down criterion. The value judgment about the performance in this test will thus be passed on the basis of the performance shown by an individual in reaching or not reaching to the set standard and attempt will always be made to help the students to attain mastery level (enabling him to solve all the given mathematical problems or at least as per criteria or standard laid down within the specified time with specified accuracy and method of presentation). Here in passing the necessary value judgment over the performance of an individual no attention is paid over the performance of others in his group.

In comparison to such criteria based evaluation, norm referenced evaluation concentrates on interpreting pupil's achievement in relation to the achievements of other pupils in the class group, school, state or country. Here the typical performance or the norm of a group is used as the basis for judging individual student's performance. Thus the value judgment over the performance of an individual student is passed in relation to the performance of other students in the group. His performance is said to be satisfactory or unsatisfactory on the basis of the comparison made with the achievements of other students in his class or group.

The distinction between these two types of evaluation can be made more clear with the help of the following given comparison table.

Table

Comparison between norm-referenced and criterion-Referenced evaluations

1.	Here interpretation of a pupil's performance is based on his relative standing in some known group i.e. class, school, state etc. (e.g. stood better than 80 percent of the students of his class)	Here interpretation of a pupil's performance based on a set standard or criterion of performance (e.g. solves 9 out of the 10 given additional problems without error).
2.	It is quite helpful in comparing the individual pupil's performance with the performance of other pupils in his group or class.	It focuses on comparing the individual pupil's performance or achievement with the set standard or criterion of performance rather than trying to compare it with other's performance.
3.	It helps in discriminating students in relation to their relative standing in the group.	It helps in discrimination and describing the pupil's abilities and inabilities to attain the set standard or criterion of good performance.
4.	Tests used at the end of instruction (summative evaluation) typically emphasize norm-referenced measurement.	Tests used during instruction (Formation evaluation) typically favour criterion-referenced measurement
5.	Such type of evaluation is mainly used in the survey type of research work, providing division and grades to the students and comparing the achievements of the sections, schools districts and states in the country.	Such type of evaluation is mainly used for motivating and helping the individual students to reach the mastery level, or providing individualized instructions and remedial teaching.
6.	Here intended outcomes are seldom specified in terms of expected performance prior to test construction.	Here it is quite essential to specify clearly the expected level of performance i.e. the desired level of behavioural changes prior to the cons traction of the test.
7.	Norm-referenced evaluation is responsible for unhealthy competitions and rivalries and therefore leads to so many evils and injurious to low scoring students.	Criterion-referenced evaluation helps and inspires a pupil, to compete with none other than one's self. He has a criteria and mastery level before him for improving his progress on the learning path.

8.	Norm-referenced evaluation represents the age old classical system of appraisal and measurement with an eye to classify and describe the performance of an individual by comparing it with others in the group.	Criterion-referenced evaluation represents the new trend in the process of appraisal and measurement involving individualization of instruction, sequencing of learning, the development of programmed material and the concept of mastery learning.

Check Your Progress

Notes : a) Write your answers in the space given below

b) Compare your answer with the one given at the end of the unit

1. Write a short note on evaluation.

   ……

   ……

2. List out the purpose of evaluation.

   ……

   ……

3. State any three objectives of evaluation.

   ……

   ……

4. What do you mean by criterion referenced evaluation?

   ……

   ……

11.6 ACHIEVEMENT TEST

The term achievement is often understood in terms of pupil's scores on a certain school test. If, for instance, a student is tested in two school subjects, say Language and arithmetic and in one subject he gets 70% marks while in the other 60% marks, it is understood that his achievement in language in which he gets 70% marks is better that in arithmetic in which he gets 60%. This is a loose way of understanding the concept of achievement. More intelligently understood, achievement means ones learning attainments, accomplishments, proficiencies, etc., achievement is directly related to pupils growth and development in educational situation where learning and teaching are intended to go on.

The concept of achievement involves the interaction of three factors, namely, aptitude for learning, readiness for learning and opportunity for learning. Besides these factors, the concept involves health and physical fitness, motivation and special aptitude, emotional balances and unbalances. Achievement in education, precisely speaking, implies ones knowledge, understanding or skills in a specified subject or a group of subjects.

Achievement test constitute an important tool in school evaluation programme. It is necessary for the teacher to know how far the pupils have attained in a particular subject area. Pupils differ in their attainments. In the school evaluation programme, various forms of achievement tests are used to measure the extent of learning of the pupils.

"Any test that measures the attainment or accomplishments of an individual after a period of training or learning is called an achievement."-N.M Dowinie.

"An achievement or proficiency test is used to ascertain what and how much has been learnt or how well a task can be performed, the focus is on evaluation of the past without reference to the future, except for the implicit assumption that acquired skills and knowledge will be useful in their own right in the future."-Super.

Achievement tests are useful aids for diagnosing a student's specific learning needs, for identifying his relative strengths and weaknesses, for studying his progress and for predicting his success in a particular curriculum"-Waters.

Of all the different types of examinations, achievement tests are used most frequently. Achievement tests differ from intelligence or aptitude tests in that – former measures the quality and quantity of learning attained in a subject of study or group of subjects after a certain period of instruction, the later measure pupil's innate capacity for attainment or accomplishment independent of any learning. These tests predict performance in a certain subject or group of subjects.

11.6.1 Functions of Achievements Tests

The major functions of achievement tests are:

- To provide basis for promotion to the next grade.
- To find out at the beginning of a year where each student stands in the various academic areas.
- In many schools, a certain grade or class has a number of sections. Achievement tests help in determining the placement of a student in a particular section.
- A teacher can use achievement test to see for himself how effectively he is doing, what is getting across pupils and what is not.
- To motivate students before a new assignment is taken up.
- Achievement tests expose pupils difficulties which the teacher can help them solve.

- ❖ To report to the parents the place of a student in a particular section according to the achievement scores.
- ❖ To diagnose a student's specific learning needs, relative strengths and weakness.
- ❖ To predict future progress and for predicting his success in a particularly curriculum.
- ❖ To reflect teachers effectiveness.

11.6.2 Advantages to the teachers and administrators

- ❖ Through achievement tests, the teacher can know the general range of abilities of students in the class.
- ❖ The teacher can select appropriate materials of instruction.
- ❖ The teacher can determine and diagnose the strengths and weaknesses of students in various subjects.
- ❖ The teacher can find out gifted and backward children.
- ❖ Tests help to discover backward children who need help and plan for remedial instruction for such students.
- ❖ Tests help to select talented pupils for providing enhanced curriculum.
- ❖ Through test, teachers can select students for the award of special merits or scholarships.
- ❖ Tests help to evaluate the extent to which the objectives of education are being achieved.
- ❖ They help to discover the type of learning experiences that will achieve those objectives with the best possible results.

11.6.3 Types of Achievement Tests

Achievement tests are of two main types:

- ❖ Teacher-made achievement tests and
- ❖ Standardized achievement tests.

Teacher made achievement tests may further be divided into two categories.

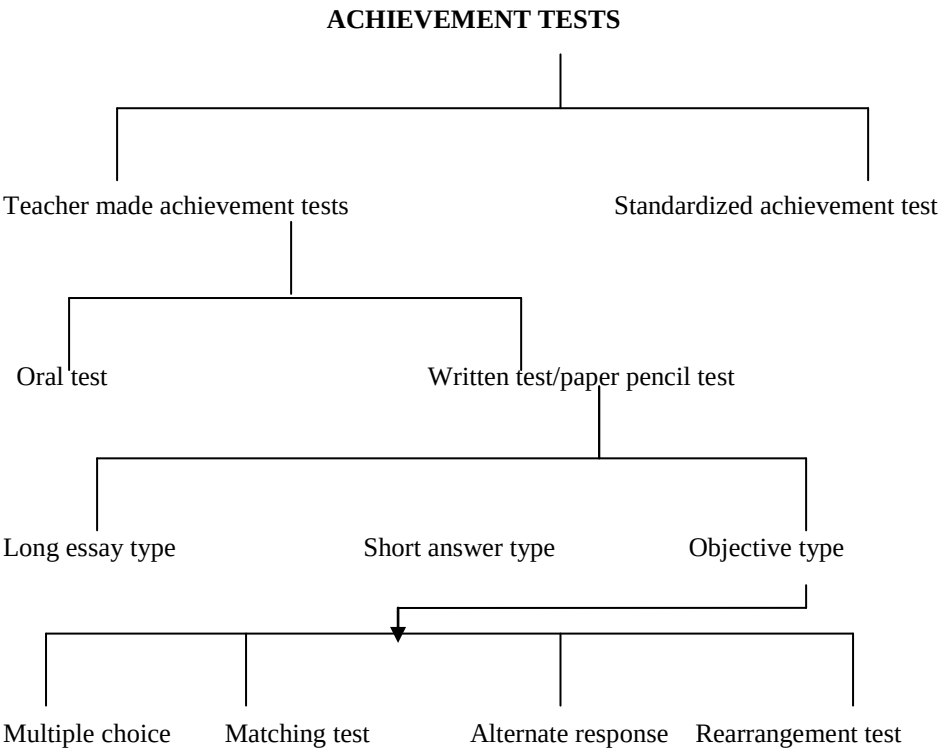
1. Written or paper and pencil test.
2. Oral tests.

Written tests can still further be classified as:

(i) Long essay type

(ii) Short answer type

(iii) Objective type



11.7 Construction of Achievement Test

A test can be made an effective instrument of evaluating the achievement of objectives, the content and the learning activities. We should not evaluate only the content; we have to evaluate the total behaviour of the pupils. For this, the achievement of the objectives of all the three domains (cognitive, affective and psychomotor) has to be evaluated.

Tests can be conducted at different times during a course.

They are:

- ❖ At the end of teaching a daily lesson;
- ❖ At the end of teaching a unit;
- ❖ At the end of the term;
- ❖ At the end of the year or curriculum;

The time factor for testing depends upon the nature of the objectives to be tested. Knowledge objective can be suitably tested either in the course of teaching or at the end of teaching a lesson. The objective of understanding or comprehension requires a comparatively longer period for its testing. Objectives of application and skill require a sufficiently longer period for their testing. The long ranged objectives, such as interest, attitude and appreciation take a lot of time for their development. These objectives can well be tested at the end of the year or when the curriculum has been completed.

Teachers should exercise their discretion in testing the objectives, keeping n mind the nature of objectives in relation to the time factor. They should change the weight age of the objectives to be tested at different times.

Testing, if properly done, leads to good learning. Testing should be pre-planned, systematic and scientific. Planning of lessons will give the teachers a full idea of how much weight age is to be given to the content, the objectives, the form of questions, etc., while planning the tests.

The preparation of a good test is a systematic process having well defined stages. The important steps envisaged in the preparation are as follows:

1. Planning of the test
2. Writing the test items
3. Reviewing and editing
4. Arranging the items
5. Providing directions
6. Preparing the scoring key and marking scheme
7. Administering and scoring the test

1. Planning of the test

Test planning encompasses all of the varied operations that go into producing the test; but it must also involve careful attention to item difficulty, to type of items to directions to the examiner.

a. Preparation of design

Designing is the first and most important step in the construction. It is at this stage that we plan to build in the test important qualities-Validity, Reliability, objectivity and practicability. In order to accomplish that, the test constructor has to take a number of decisions regarding the selection of objectives, content, form of questions, the difficulty level of test items and the Weightages to be allotted to the objectives to the comment and the forms of questions.

b. Identification of the objectives and allotting weightage to the objectives

Identification of the instructional objectives and stating them in terms of specific observable behaviors. After the objectives are identified and stated the test maker has to decide the relative weight in the test. The important fundamental principal to be observed here is that the test should reflect the actual emphasis being given to various mental process enduring instruction. There cannot be any cut and dried formula for assigning weights to various objectives.

Table showing the weightage allotted to the objectives

S.No	Objectives	Marks Allotted	Percentage
1	KNOWLEDGE	5	20
2	UNDERSTANDING	8	32
3	APPLICATION	10	40
4	SKILL	2	8
	TOTAL	25	100

c. Selection of the content and allotting Weightage to the content

It becomes very necessary to decide the weights to be given to different parts of it. As the whole syllabus cannot be covered through any single test a convenient number of units can be selected for testing. When this is done a decision about the weights to be given to those units has to be taken so as to represent the actual emphasis on them in instruction. In assigning relative weights to units a number of factors will have to be taken into account. How important is the unit in the total learning experience? How much time was devoted to it during instruction? Although there are a number of such considerations, the easiest method is to decide the weightages on the time required to teach various units.

Example

Table indicating the weightage given to three units namely polynomials, functions and quadrilaterals.

S.No	Unit	Marks given	Percentage
1	Polynomials	10	40
2	Functions	8	32
3	Quadrilaterals	7	28
	TOTAL	25	100

d. Selection of the form of question and giving weightage to the questions

Decide about the form of questions to be used, the number of questions to be chosen and the relative weightage and to be given to each form.

Example

The weightages to different forms of questions could be as follows.

S.No	Form	Marks given	Percentage
1	Long answer (LA)	6	24
2	Short answer (S.A)	9	36
3	Objective type (O)	10	40
	TOTAL	25	100

e. Distribution of difficulty level

A decision also has to be taken concerning the distribution of difficulty level. The distribution of difficulty level in a test will depend upon the purpose of the test as also on the group of students for whom it is designed. To get optional discrimination through a test most of its question should be of average difficulty level. A few questions here and there may be easy and difficult.

Example

S.No.	Difficulty level	Percentage
1	Difficult questions	15
2	Average questions	70
3	Easy questions	15
	Total	100

f. Preparation of Blue print

Preparation of the Blue print refers to the final stage of the planning of a test. It is a three dimensional chart showing the weightage given to the objective, content and the form of questions in terms of marks. It is also called as a table of specifications as it related outcomes to the content and indicates the relative weight given to each of various areas. The blue print helps to improve the content validity of teacher made tests. It defines as clearly as possible the scope and emphasis of the test; it relates objectives to the content it provides greater assurance that the test will measure learning outcomes and course content in a balance manner.

Objectives	Knowledge			Understanding			Application			Skill			Total
Content	L.A	S.A	O	L.A	S.A	O	L.A	S.A	O	L.A	S.A	O	
Polynomial			1(2)		3(1)	1(1)		3(1)	1(1)				10
Functions			1(2)		3(1)	3(1)			1(2)				8
Quadrilateral			1(1)				4(1)			2(1)			7
Total	5			8			10			2			25

The major and final responsibility will be that of the teacher, who the decision maker is. To be of the utmost benefit, it should be prepared well in advance. It would thus assist the teacher in organizing his teaching material, serve as a monitoring agent, and help keep the teacher from straying off his instructional track.

The number inside the bracket indicates the number of questions and the number outside the bracket indicates marks allotted to each questions.

Scheme of options :(choices)

There are two types of options:

They are external options and internal options

In case of external options there are various methods.

- The students are asked to attempt any six out of the given ten questions. This is over all options.
- The Students are asked to attempt any two from the first section, any four from the second section and any fiver from the third section. This is section wise options.
- The students are asked to answer either one question or the other alternate question. This is questions wise options.

In case of internal options, the option is given within a question for example: write an essay on any two of the following.

Options should not be given because the purpose of a unit test is to have complete evaluation of the achievement of objectives of the content and the learning experiences. This purpose is not served if options are given. It is extremely difficult to set two alternative questions or two alternative groups of questions, which measure the achievement of the same objective, the same content or the same learning experiences. The scheme of options may develop some wrong practices among students. They may omit some content. The scheme of options in a test paper influences even the teachers training.

Sections in the question paper

Depending on the type of question, the question paper can be divided into sections. One section can include essay type questions, another section short answer type and the third section objective types. Separate marks for each section should be mentioned. For each section, separate instruction also has to be given.

2. Writing the test items

Write the test items according to the table of specifications. He should take up each cell of the blue print and draft and item taking care

of the various dimensions the objective, the content and the form as laid down in the blue print.

While teaching a lesson, the teacher can prepare questions also. Continuously this work has to be done. When the teacher asks questions in the class room or when the teacher conducts a test, the students respond to them. From the responses of the students multiple choice items can be coined.

Depending on the concept, the teacher teaches in the class, different types of questions can be prepared in the sub concepts. The same item can be presented in different forms and kept ready to be used at any time. So question banks should be prepared by the teacher of the same subjects. Newly prepared questions can be added to the existing questions in the question bank every day, every month and every year.

If an item bank is ready and if a blue print is ready, the only job that remains to be done by the teacher is to select test items form the item bank in accordance with the design of the blue print. A lot of effort is needed to coin better test items. It calls for a mastery of the content and practice in wording test item. But once they are ready, editing a question paper will become easier.

Grouping of test items

While grouping test items, the following points are to be considered.

- ❖ All the objective type items should be grouped in one sections, while the short answer in the next section and essay type items should be grouped in another sections.
- ❖ In the section of objective type items, the items having the same format, examples yes-no type, true-false type, matching type should be grouped together.
- ❖ Items in each section should be arranged in order of their difficulty, as far as possible.

Sections in a question paper

Generally, objective type items are grouped under section A and short answer type questions under section B essay type questions under section C.

3. Reviewing and editing

The pool of items for a particular test after being set aside for a time can be reviewed by the help of experts. The following are to be considered:

1. Does each item measure an important learning outcome included in the table of specification?

2. Is each items appropriate for the particular learning outcome to be measured?
3. Does each item present clearly formulated test?
4. Is the item stated in simple, clear language?
5. Is the difficulty of the item appropriate for the students to be tested?
6. Does each item fit into one of the cells of the blue print?

4.Arranging the items

The items should be arranged so that all items of the same type are grouped together. The items should be arranged in the order of increasing difficulty. It may be desirable to group together items which measure to same learning outcome or the same subject matter content.

5.Providing directions

The direction should be simple and concise and yet contain information concerning each of the following:

Purpose of the test, time allotted to complete the test, how to record the answers, whether to guess when in doubt about the answer and marks allotted for each question. General instructions may be given at the beginning of a question paper.

E.g. This paper consists of A,B, and C sections.

- ❖ Answer any two from section A: answer any four form section B and all questions are compulsory in section C.
- ❖ Specific instruction related to each section may be given at the beginning of each section.

e.g: Answer to section A should be given on the question paper itself.

e.g: Answer to each question under section B should not exceed one page.

6.Preparing the scoring key and marking scheme

When the test has been assembled and ready to administer, it becomes necessary to prepare some other important accessories in the form of a scoring key for objective type questions and marking scheme for supply-type questions, such as short answer and essay type questions.

Scoring key refers to the prepared list of answers to a given set of objective type questions. The examiner simply compares the answers given by the students with these in the scoring key and thus arrives at the marks to be awarded to the students.

A marking scheme is essential in the case of short answer and essay type items; the following are the important items in as scheme.

- Points or steps expected in the answer.
- Description of each point or step expected in the answer.
- The weightage to each of these points or steps.

Advantages

❖ A marking scheme helps the examiner to bring about a uniformity of standard in assessing and there by increases the objectivity of the test.

❖ Many examiners may be judiciously involved in assessing answer books. It will be of help to them.

A scoring key is essential in the case of objective test items. In the scoring key, the itemwise correct response in terms of its number is to be mentioned.

Marking scheme in mathematics will be desirable not only to analyse the solution into important stages and to distribute marks over item but each stage may be looked upon from the point of view of the method involved as also from expected accuracy. The marks for each stage therefore may be divided into two component; marks for the method and those for accuracy.

7. Administering and Scoring the Test

At this step, it is important to make sure that all students know exactly what is expected of them and to provide them with the most favourable conditions for taking the test. After the administration of the test, the scoring can be done with the help of the scoring key and marking scheme.

Check Your Progress

Notes : a) Write your answers in the space given below
b) Compare your answer with the one given at the end of the unit

5. Define achievement test

……
……

6. Mention any four functions of achievement tests

……
……

7. What are the two main types of achievement tests?

……

8. List out the steps involved in the construction of achievement tests.

……

11.8 LET US SUM UP

For measuring and judging various kinds of learning in mathematics, there are a variety of tools and techniques. A teacher has to devise and administer a variety of tools and techniques for evaluation. This unit attempted to deal with various forms of tools and techniques for evaluation that are used to measure the accomplishment of the pupil's and the role of evaluation in the teaching and learning of mathematics and various types of evaluation.

Achievement test constitute an important tool in school evaluation programme. It is necessary for the teacher to know how far the pupils have attained in a particular subject area. Pupils differ in their attainments. In the school evaluation programme, various forms of achievement tests are used to measure the extent of learning of the pupils.In this unit, functions of achievements tests, its advantages to the teachers and administrators, types of achievement tests and construction of achievement test have been dealt in detail.

11.9 UNIT-END ACTIVITIES

1) What is evaluation? Throw light on its various types and their specific uses.
2) Compare Norm referenced and criterion referenced tests.
3) What are the major functions of achievement tests?
4) Describe various types of achievement test.
5) List the steps involved in construction of an achievement test.

11.10 ANSWERS TO CHECK YOUR PROGRESS

1. Evaluation in general is an act or a process that allows one to make a judgment about the desirability or value of a measure. Evaluation in educational situation is thus a relatively new term introduced to designate a more comprehensive concept of measurement than is implied in conventional tests and examinations, the emphasis in evaluation being upon broad personality changes and more objectives of an educational programme and therefore include not only subject matter achievements, nit also attitudes, ideals, ways of thinking, work habits and personal and social adaptability. Thus evaluation is not just a testing programme. Tests are but one of the many different techniques that may contribute to the total evaluation programme.
2. Purpose of Evaluation

- ❖ To provide information for grading, reporting to parents and promoting students
- ❖ To evaluate the effectiveness of a single teaching method or to appraise the relative worth of several methods.
- ❖ To evaluate the entire educational institution and to show how several of its aspects could be improved.
- ❖ To collect information for effective educational and vocational counseling.

3. **Objectives of Evaluation**
   - To help in laying out or bringing modification in the instructional and educational objectives of teaching mathematics for a particular grade in a specified teaching-learning situation.
   - To help in the selection and organisation of learning experiences for the teaching of mathematics related to a particular grade and teaching-learning situation.
   - To help in the selection and use of general and specific teaching-learning strategies and methods for the teaching of specific topics and branches of mathematics in a particular grade and teaching-learning situation.
4. In criterion referenced evaluation, the tools of evaluation, tests and techniques are so constructed as to measure student learning according to a predetermined standard of achievement or performance. In other words, evaluation measure or a test based on reference to a "criterion" *i.e.* specific teaching-learning objective fixed in advance may be referred to as a criterion referenced evaluation.
5. "Any test that measures the attainment or accomplishments of an individual after a period of training or learning is called an achievement."-N.M Dowinie.
6. The major functions of achievement tests are:
   - ❖ To provide basis for promotion to the next grade.
   - ❖ To find out at the beginning of a year where each student stands in the various academic areas.
   - ❖ In many schools, a certain grade or class has a number of sections. Achievement tests help in determining the placement of a student in a particular section.

- ❖ A teacher can use achievement test to see for himself how effectively he is doing, what is getting across pupils and what is not.

7. Achievement tests are of two main types:
   - ❖ Teacher-made achievement tests and
   - ❖ Standardized achievement tests.
8. The important steps envisaged in the preparation achievement tests are as follows:
   - Planning of the test
   - Writing the test items
   - Reviewing and editing
   - Arranging the items
   - Providing directions
   - Preparing the scoring key and marking scheme
   - Administering and scoring the test

11.11 SUGGESTED READINGS

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UNIT – XII

ASSESSMENT AND EVALUATION -II

Structure

12.1 INTRODUCTION

The diagnosis of student deficiencies in mathematical achievement is one of the most difficult tasks that confront teachers in their day-to-day life. Diagnosis in education resembles diagnosis in medicine. The aids to diagnosis are known as diagnostic test. These tests are used to obtain evidence of pupil difficulties. It serves as a basis to the teachers for improving instructional methods, instructional material and learning procedures. Therefore this unit deals with the need, construction and uses of diagnostic test.

As in Medical field, Doctors after diagnosing, give medication, medicines as remedy, so in educational field, remediation should be provided. So the teachers should have the knowledge about Remedial Teaching in Mathematics.

12.2 OBJECTIVES

At the end of the unit, the student-teachers will be able to

- Know the meaning of diagnostic test.
- Understand the purpose of diagnostic test.
- Describe the steps involved in the construction of a diagnostic test
- Prepare diagnostic test.
- Recognize the meaning and purposes of remedial teaching.
- Suggest remedial teaching.
- Discuss the importance of prognostic test.

12.3 DIAGNOSTIC TESTS

12.3.1 Need for Diagnostic Test

Most standardized and teacher-made achievement tests are designed to give an indication of how far the student has progressed towards the accomplishment of specific objectives measured by the test. These objectives however are grouped in broad categories. They cover a broad area and result in a total score which reflects overall achievement in the area tested. Thus the teacher can say that a pupil is doing well in arithmetic or poorly in arithmetic, but they do not know why, nor do they know what are the concepts causing difficulty. It will identify students who are having relative difficulty in an area, but it will not identify the causes of the difficulty. Such survey (achievement) tools serve a useful function, but in order to help the student with a disability, the teacher will need to analyse the nature of the difficulty and the causes for the trouble.

There are tests which have been devised to provide information about the specific nature of pupil's difficulties in given subject areas. These tests are called diagnostic tests. Any test can be used as a diagnostic test in a limited way by examining students' performance in the individual items which make up the test rather than on the test as a whole. Thus diagnostic tests measure somewhat narrower aspects of achievement than survey tests. In other words, diagnostic tests yield measures of highly related abilities underlying achievement in a subject. They are designed to identify particular strengths and weaknesses on the part of the individual child and within reasonable limits to reveal the underlying causes.

The diagnostic test attempts to break a complex skill like computation into related parts such as addition, subtraction, multiplication and division and to provide separate measures of these sub-skills. Such measures can help the teacher to locate the sources of difficulty using which constructive action can be taken.

12.3.2 Construction of Diagnostic Tests

The steps involved in the construction of a diagnostic test are as follows

- Identification of the problem areas
- Detailed content analysis
- Listing all the learning points.
- Arranging the learning points in the logical sequence
- Writing test items. (preferably two or three items of free response type) for each learning point
- Clubbing the items around the learning points.
- Providing clear instructions
- Preparing a scoring key and a marking scheme.
- Providing the time limit as required by individual students.
- Administration of the test.

After administering the test, the following procedure may be followed for analysing the performance and identifying the weaknesses.

- Item-wise analysis of the performance of each student.
- Qualitative and quantitative analysis for identifying the strengths and weaknesses.
- Identification of the causes for learning difficulties.
- Preparation of a diagnostic chart for each student.
- Planning and implementing highly individualised remedial programmes
- Evaluating the effectiveness of the programme.

12.3.3 Uses of Diagnostic Tests

The following are the uses of diagnostic tests.

The diagnostic tests

- point out inadequacies in specific skills.
- locate areas in which individual instruction is required
- furnish continuous information in order that learning activities may be most productive of desirable outcomes.
- serve as a basis for improving instructional methods, instructional material and learning procedures.

Check Your Progress

Notes : a) Write your answers in the space given below

b) Compare your answer with the one given at the end of the unit

1. What is diagnostic test?

……

……

2. Mention the need for diagnostic test.

……

……

3. State the uses of diagnostic test.

……

……

12.4 REMEDIAL TEACHING IN MATHEMATICS

12.4.1 Meaning and Purpose

The term remedial teaching as the name suggests stands for the teaching or instructional work carried out to provide remedial measures for helping the students (or individual student) in getting rid of their common or specific weaknesses or learning difficulties diagnosed through diagnostic testing or some other measures earned out for such diagnosis.

Diagnosis thus provides a solid base for hypothesizing the general and specific causes underlying the weaknesses or learning difficulties of the students of a class/group (or a particular student). It is thus true that as the diagnosis so is the remedy for the removal of the difficulty. Let us think over the possible remedial measures in the light of this very assumption.

- In case the class as a whole is demonstrating a particular type of weakness or learning difficulty then the treatment should be followed by a common remedial programme. For this purpose, special classes may be organised or special methods and techniques may be employed for making the class understand properly the concepts or skills etc. related to the area of weakness.
- In case the weakness or learning difficulties diagnosed are of specific nature applicable to the individual students. Then the treatment should also be individual specific. Mathematics is a quite sequential subject. Here the study of a topic is dependent upon the previously learnt topics. In case the student is lagging behind in the learning of the pre-requisites essential for the study of the topic in hand, then the treatment in terms of the remedial teaching must necessarily be given

for making up the previous deficiencies in learning. The child should be helped individually for this purpose by making use of the methods, techniques and teaching-learning situations best suited to the individuality as well as nature of the content.

- In case the learning difficulty felt at present is not the product of the deficiencies in terms of pre-requisites or previous learning, then its roots must necessarily lie in the present set up of the teaching learning. The results of the diagnostic testing and adequate analysis of the errors committed by the individual student then may bring on the surface some or the other probable causes for the diagnosed weakness and learning difficulty of the individual students. In the light of such diagnosis a proper remedial teaching programme should be taken into hand.

It attempts to find a procedure which causes child to correct his errors of skill and thought. It is used to correct

- Emotional difficulties
- Subject difficulties
- Teaching/learning difficulties

As in Medical field, Doctors after diagnosing, give medication, medicines as remedy, so in educational field, remediation should be provided.

12.4.2 Features of Remedial Teaching:

- Follow psychological approach
- Individualistic in nature
- Motivational in nature
- Continuity maintained
- Interestful
- Innovative mode
- Establish positive attitude in learners mind

12.4.3 Methods used for Remedial Teaching:

- Case study method
- Orientation service
- Individual instruction
- Group instruction
- Play way approach

- Situational remedy
- Innovative technique
- Environmental study
- Discussions
- Expert talk
- Better parent-teacher interaction
- Stimulating interest and thoughts of students

12.4.4 Remedial Teaching in Mathematics

- Drill work: If students are weak, in some formulas, law, let them solve problems based on it. Problems should be having variety and good number of problems, should be given to students.
- Using innovative materials in classroom: Teacher should use different instructional materials, to appeal to senses of learners mind and to bring abstractness of concept to zero. Make use of different teaching aid at introductory, developmental and evaluation stage to make concept more clear.
- Using play-way technique: Different geometric figures like square, kite etc. could be conceptualised in class through different games. Bring quiz programmes, Antakshari, riddles, mathematical jokes, mathematical poems, spatial relation in your class, so that students learn difficult subject at ease.
- Group discussions: Let there be group discussions. Shy and weak students will also gather the confidence to talk and their difficulties will be shared in public. Also, lot of information will be transferred from one cognitive class to another, resulting in removal of abstractness.
- Activity method: Students face difficulty in subjects like money transactions, conversion, doing operations on fractions etc., so teacher can assign a project work, where students do lot of references and go out of class room to solve the problem.
- Rapport with parents: Mathematics teacher should have good rapport with students and their parents. Parents should always be made aware of student's weak points so that even they can take some kind of home work to make concept easy. Parents can speak in their mother tongue and students can have mastery over concept, by studying at peace in their houses too. The teachers will supplement the part in schools.

12.5 PROGNOSTIC TESTS

One of the important uses of tests is predict how individuals behave in certain situations. For example, when tests are used to select students for colleges, the major purpose is to forecast how well students will do when they actually go to college. One group of aptitude tests is made up of tests designed to predict readiness to learn or probable degree of success in some subject or segment of education .These are called prognostic tests. In other words, prognostic tests are designed to predict the student's ability or readiness to undertake the study of a school subject.

Prognostic tests are intended for use in prognosis or prediction of future success in specific subjects of the school curriculum. As they usually test the background skills and abilities found to be prerequisite for success in the particular subject, prognostic tests are more common among subjects in which success can be rather well defined in terms of certain basic abilities. They also frequently test some of the aptitude factors that are not directly dependent upon previous training of a specific topic. Therefore, prognostic tests, probably most closely related to aptitude tests, but not unrelated to inventory tests, are more properly classified as educational tests, than as intelligence tests ,through they unquestionably do measure certain special aspects of intelligence.

Check Your Progress

Notes : a) Write your answers in the space given below

b) Compare your answer with the one given at the end of the unit

4. Write a short note on Remedial teaching.

……

5. List any four methods used for Remedial Teaching.

……

6. Mention any three features of Remedial Teaching

……

7. What is prognostic test?

……

12.6 LET US SUM UP

Diagnosis thus provides a solid base for hypothesizing the general and specific causes underlying the weaknesses or learning difficulties of the students of a class. It is thus true that as the diagnosis so is the remedy for the removal of the difficulty. In this unit, the Need, Construction and uses of diagnostic test and features and methods

remedial teaching have been dealt in detail. The importance and use of prognostic test has also been highlighted.

12.7 UNIT-END ACTIVITIES

1) List the steps involved in construction of diagnostic test.
2) What is the need of diagnostic testing in mathematics? Discuss its features.
3) What is remedial teaching? Describe various techniques / measures, which can be used in remedial teaching.
4) What is prognostic test? Throw light on its specific uses.
5) Compare diagnostic test and prognostic test.

12.8 ANSWERS TO CHECK YOUR PROGRESS

1. There are tests which have been devised to provide information about the specific nature of pupil's difficulties in given subject areas. These tests are called diagnostic tests.
2. The diagnostic tests are designed to identify particular strengths and weaknesses on the part of the individual child and within reasonable limits to reveal the underlying causes.
3. The diagnostic tests
    a. point out inadequacies in specific skills.
    b. locate areas in which individual instruction is required
    c. furnish continuous information in order that learning activities may be most productive of desirable outcomes.
    d. serve as a basis for improving instructional methods, instructional material and learning procedures.
4. The term remedial teaching as the name suggests stands for the teaching or instructional work carried out to provide remedial measures for helping the students (or individual student) in getting rid of their common or specific weaknesses or learning difficulties diagnosed through diagnostic testing or some other measures earned out for such diagnosis.
5. Methods used for Remedial Teaching:
    - Case study method
    - Orientation service
    - Individual instruction
    - Group instruction

6. Features of Remedial Teaching:
    - Follow psychological approach
    - Individualistic in nature
    - Motivational in nature
7. Prognostic tests are designed to predict the student's ability or readiness to undertake the study of a school subject.

12.9 SUGGESTED READINGS

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UNIT - XIII
TEACHING FOR PERMANENCE

Structure

13.1 INTRODUCTION

A good and successful teacher cannot afford to leave the minds of his students inadequately informed. Mathematics is more a skill subject than a knowledge subject. A skill cannot become one's own unless it is practiced periodically. Mathematical principles, working rules and formulae should get fixed in the minds of students and they should have a meaningful understanding of their applications.

Skills need to be perfected and maintained through systematic drill. Concepts and relationships must be reviewed and applied at frequently recurring intervals.

This unit discusses about drill and review which are the means to strengthen the knowledge already acquired.

Motivation will enable the students to learn diligently and purposively. Therefore, suitable means of motivation should be employed to ensure that the teaching is impressive.

This unit brings out various means of motivation, rationalization and concretization which helps for better understanding of the mathematical concepts and effective learning. This unit provides a knowledge of correlation of mathematics which can enhance the effectiveness of instructional process by relating mathematics to other areas of interest.

Also this present unit equips the student teacher with adequate knowledge about individualized instruction which caters to individual differences.

13.2 OBJECTIVES

At the end of the unit, the student-teachers will be able to

- Understand the functions and advantages of drill work.
- Describe the role of review in mathematics learning.
- Differentiate between drill and review.
- Know various means of motivation.
- Understand the importance of rationalization and concretisation for effective learning and understanding of mathematical concepts.
- Explain the correlation between mathematics and other subjects.
- Understand the characteristics of individualized instruction.
- Know some suggestions for making home assignments more effective.

13.3 DRILL

Any subject matter, however, is likely to be forgotten, no matter how well it has been initially mastered unless it is maintained by repeated application and practice. This is, especially true of mathematical skills and relationships. Skills need to be perfected and maintained through systematic drill. Concepts and relationships must be reviewed and applied at frequently recurring intervals. Drill and review are means to strengthen the knowledge already acquired. Drill and exercises have a valuable place in the teaching of mathematics.

The Functions of Drill

The place of drill inteaching of mathematics has been a controversial issue in recent years. There are two viewpoints regarding

the use of drill in mathematics as being futile and without value.

- Mechanical drill helps in rapid memorisation of details and the automatization of process to the extend of neglecting meanings. The proficiency acquired without understanding is not likely to endure long.
- Meanings alone have value, that the development of new concepts and understanding is all that matters, and that drill, in its restricted sense has no place in the educational process.

These points of view overlook the important element of fixation without which it is impossible to organize and relate concepts. Neither is it possible to carry on any process at a reasonable level of efficiency. Meaningful drill and exercises could yield fruitful outcome.

Drill and Exercise

- Help in the fixation of rules, principles and formulae already acquired by the students.
- Strengthen the connection between related concepts.
- Provide essential means of attaining some of the desired controls
- Help in increasing speed and accuracy in solving mathematical problems as the learners become proficient in mathematical skills and operations.
- Help in the automatization of mathematical facts, formulae, operations and skills through systematic and repeated practice
- Help in habit formation
- Help in exhibiting greater efficiency on the part of the students to have a command over the tools of knowledge that they have acquired
- Help in improving self-confidence through a sense of achievement
- Provide stimulus for further learning
- Provide encouragement to slow learners and average students

It should be clearly remembered that a drill is not mere repetition of an act to acquire perfection in the performance. The needs and interests of the students are always to be taken into consideration. If instruction is to be valuable, understanding must go hand in hand with operational facility. Students should not be drilled on procedures that they do not understand. Such drill lacks significance and motive. A drill should be intelligently planned so that it accomplishes its purpose; otherwise it may be a sheer waste of time and energy.

Principles of Drill

Some important considerations relating to drill in mathematics have been enumerated.

- A drill must have some purpose. In other words a drill must be specific. It must aim at automatizing a specific response/skill or eliminating erroneous responses.
- A drill to be most effective, must be sufficiently and properly motivated and once initiated it should be well organised
- Drill material must contain intrinsic interest so that the students can see value in it. When they recognise something as important or interesting in itself, they will work with enthusiasm and concentration and their work will be more effective.
- Drill materials must be highly individualized so as to meet the individual needs and abilities.
- The proficiency to be attained must be defined. The degree of proficiency must be according to the purpose and need of the individual child.
- Drill exercises should contain enough material to keep all the students profitably occupied throughout the drill period.
- Drill periods should be of short duration. Long periods of continuous drill become tiresome and ineffective. In general, no drill period should exceed more than 20 minutes. If it requires more time to master the desired point of proficiency, smaller periods of practice at recurring intervals can be provided.
- When a drill is given, importance should be given to accuracy, and for the time being, speed should be regarded as of secondary importance. Therefore, it is essential to supervise closely the initial work of the students on any new process so as to avoid fixation of wrong processes.
- The method of applying mathematical checks can be used in connection with drill work just as it can with problem work.
- Answers should be provided for the students to check the correctness of their responses.
- Wherever possible drill materials should be provided with some means by which the student can score his own work and compare his performance with an established standard.
- The learner should both understand what he is practising and appreciate its significance to him as an individual .

- A drill should follow the development and discovery stages of learning and be used to reinforce and extend basic learning.
- Drill exercises should be varied so that it does not become monotonous for the students.
- A drill should be used when and where needed.
- It should not be used as a punishment nor should things already well learned be assigned for more practice.
- In general it is better to provide drill upon the whole process rather than parts thereof, unless some particular parts need practice.
- Mechanical repetition and rote memorization should be avoided by stimulating thinking and insight.
- Drill exercises should be sufficiently diversified to provide worthwhile and stimulating practice for students of different attainments and capacities.

Demerits of Drill

In spite of the various advantages, if a drills is not planned and executed properly, it can lead to:

i. Mechanical reproduction of ideas.
ii. Rote memorization of principles and formulae
iii. Monotonous and routine work and
iv. Learning without clear understanding.

13.4 REVIEW

Review is sometimes identified with drill as both are characterized by repetition and both aim at fixations of ideas. However, a review is different from a drill as it aims not only at fixation of ideas but also viewing the material differently and in a wider perspective for better understanding of the material. It is the thoughtful organisation of the important details of the content into a coherent whole in order that relationship of the various elements to each other and to whole unit may be understood. Review emphasises thought and meaning rather than habit formation.

Advantages of Review

Review in mathematics has the following advantages.

Review helps in

a. Fixation of ideas
b. Retention of ideas
c. More effective recall of ideas

d. Thoughtful organization of the subject matter into a coherent unit

e. Better understanding of the material and ideas.

f. Systematizing and relating elements to each other.

Suggestions for effective review

a. The emphasis in review should be on thought and meaning rather than habit formation.

b. It should not involve mere repetition of facts, but meaningful reorganization of the subject matter into a coherent whole.

c. It should lead to integration of facts and ideas.

d. It should result in new learning rather than fixation of already learned ideas and concepts.

e. It should reveal the relationship among the various elements of the subject matter.

f. It can be integrated with daily classroom teaching or can be given as a specialised assignment.

g. It should provide strength and coherence to the entire structure of knowledge through continual re-association of the components.

h. Definite instructions as to how to review the lesson should be provided to the students.

Though drill and review serve some common purposes and have certain common characteristics, they differ in many ways.

Table: Differences between Drill and Review

Drill	Review
Fixation of ideas and skills through repeated practice.	Better retention of ideas and concepts through meaningful re-organisation
Relearning of already learned concepts and skills	Results in new learning
Increasing speed and accuracy through proficiency in operations	Intelligent and thoughtful appreciation of mathematical principles and skills
A rehearsal of already acquired skills	Reconstruction of ideas by establishing relationship among elements of the content
Likely to be mechanical and	Interesting and thought-provoking

boring	
Aims at automatization of relatively detailed process.	Aims not only at the fixation and retention of details but also at thoughtful organization of the important points in a unit

A teacher, while planning for teaching, should incorporate sufficient drill and review in his/ her lesson plan.

Check Your Progress

Notes : a) Write your answers in the space given below
b) Compare your answer with the one given at the end of the unit

1. Mention the demerits of drill
   ……
   ……
2. What do you mean by review?
   ……
   ……
3. Give your suggestions for effective review.
   ……
   ……

13.5 MOTIVATION

One of the important problems of mathematics teaching is that of creating and maintaining interest in the subject. This task is called, motivation. Motivation should be secured for two distinct purposes namely, following classroom teaching and taking an abiding interest in mathematics.

Why to motivate?

Motivation will enable the students, to learn diligently and purposively. The teacher has to take note of the various interests and experiences of the students. Thus the problem of motivation reduces itself to that of creating interest as well as sustaining it throughout the learning process.

In general, students are more interested in things or practical utility that in those of theoretical value, in new than in old, in those that can be linked than in those of remove appeal. But sometimes puzzling and my serious conditions have a greater attract. For them than simple and matter – of – fact ones.

Means of motivation

Effectiveness of learning is subject to two factors – understanding and responding. Suitable means of motivation should be employed to ensure that the teaching is impressive. Following are some of the means that may be employed.

1) Previous knowledge
2) Informational matter
3) Multi-sensory aids
4) Intellectual curiosity
5) Problem – solving
6) History of mathematics
7) Cultural value and
8) Recreational activities.

Motivation through previous knowledge

This is the most common form of motivation. Every teacher employs it at the beginning of a lesson as well as in the course of it. When we introduce the concept of compound interest, we make use of the knowledge of simple interest. When we introduce 'ring' in modern algebra, we recall the previous knowledge about the structure 'group'.

We use the previous knowledge for linking it with the new. The mind readily takes in the new lesson if it is seen to proceed naturally from the old. This method is particularly useful for correlating the units of a major topic or the different lessons of a teaching unit.

Motivation through informational matter

The information given may refer to the part played by mathematics in other school subjects engineering, business, trade, industry and such other fields. A person deposits under the recurring deposit scheme a particular sum for 2 years. The problem of calculating interest is to be solved by mathematics. A person wants to give a market price to an article in such a way that he should gain even after allowing some discount. This problem when discussed with students will motivate students to learn problems on trade discount. There are books and magazines which contain ample material in the role of mathematics in these fields.

Motivation through multi-sensory aid

We should employ suitable aids so as to appeal to the senses. These will enhance the effect of the teacher's oral teaching and blackboard demonstration. A clinometers will help to understand the concept of angel of elevation and depression. A geo board will help to develop many geometrical concepts. A chart where a right angled triangle is drawn with squares on each square divided into a number of

smaller equal squares will motivate Pythagoras theorem better. Films and film strips are now available for the teaching of several topics in mathematics. A chart containing pascal's triangle will help pupil find out the binomical expansions. These can be used for sustaining the interest needed in the class-room teaching and for making the impression lasting.

Motivation through intellectual curiosity

This method make use of pupil's curiosity to find out what remains to be learnt. While explaining a proof in geometry or a problem in algebra, the teacher stops abruptly at a critical point and leaves the rest of the proof as an exercise to the students, they are interested in solving it and take up the challenge gladly. Puzzles and abstract problems may arouse the interest of above-average students.

Motivation through problem - solving

The problem solving aspect of mathematics creates genuine interest in the students and sustains it. The sensing of a problem gives rise to a challenge and a state of intellectual excitement. Sometime what is given in a problem may be superfluous or insufficient. Students will take interest in finding out what more is needed to solve a problem or what unnecessary or superfluous data is given in the problems.

Motivation through references to the history of mathematics

Whenever possible, things like the historical development of the number system and the romance of measurement be used to stimulate interest. The history of π will be really interesting to students. The students will realize that mathematical truths we have today are the outcomes of great efforts spread over centuries. The life experience of great mathematicians will stimulate young minds to think that they must also strive hard and contribute to the growth of mathematics.

Motivation through emphasis on the cultural value of mathematics

The teaching of mathematics should be such that students are able to form an idea of the part played by it in social and economic changes that are taking place rapidly.

Motivation through recreational activities

Puzzles, magic squares, arrangement of numbers in a pattern etc. act as a source of motivation. Mathematics clubs can provide students with sample opportunities for further recreational activities in the form of games, competitions discussions, making teaching aids and reading of extracts from the history of mathematics and biographies.

13.6 RATIONALIZATION

The study of Mathematics develops reasoning power. Rationalization helps problem solving. Rationalization of Mathematical

processes, working rules and formulae is a must for effective learning.

In Mathematics, students have to make use of many rules which serve as a convenient tool to solve problems. Students should know how those working rules are arrived at. For instance let us take the division of 18¾ by 2 ½.

On the abstract level, this reduces itself to as follows;

18 ¾ ÷ 2 ½

75/4 ÷ 5/2

75/4 × 2/5 = 15/ 2 =7 ½.

But the students should resort to this after some such explanation as the following.

Suppose 2 ½ kg of sugar can be bought for Rs. 18 ¾, Find the cost of one kg of sugar. What should we do to find the required answer?

(Divide18 ¾ by 2 ½)

How many halves 2 ½? (5).

Can you find the cost of ½ kg (yes)

How? (By dividing 18 ¾ by 5).

What should you do to get the cost of one kg? (Multiply (18 ¾ ÷ 5) by 2). What does the original sum come to ultimately?

18 ¾ x 2/5. Now do the sum.

Very often ,if the mere working rule is taught, some students invert 18 ¾ or both 18 ¾ and 2 ½ and arrive at wrong answers.

If we develop the rational first and then arrive at the working rule. We are employing Mathematics primarily as a mode of thinking and secondarily as a tool. Generally rationalized teaching is preferable to mechanical. Rationalization helps the pupil to satisfy that there is justification for every step in his procedure and to recall the missing links in his reasoning by resorting to systematic analysis of the "if – then" pattern, retracing the steps for the purpose.

If we proceed too fast with our ambition to cover large area, learning becomes superficial. In addition to speed, we are concerned with accuracy. The objective at all times is to broaden the Mathematical understand on a rational basis. It will foster a better appreciation of the material learnt and facilitate independent thinking. It will also arouse an abiding interest in pursuing the study of mathematics.

13.7 CONCRETISATION

Concretisation helps better understanding of the Mathematical concepts. The concepts like a straight line of 'a' cms area of a square = x^2 sq. cms, area of a rectangle = lb, volume of a cube =b^3c.cms can all be, concretised through geometrical drawing. The commutative law, the associative law and the distributive law in set operations using " Union" and Intersection" can be better understood by students if they are illustrated in Venn diagrams.

Audio-visual aids help very much in concretising abstract concepts. The teacher in order to concretise abstract principles can relate them to the direct experience of the students. Concretisation is much more needed in the lower classes than in the higher classes.

Practical work helps to concretise same abstract ideas. As a rule practical geometry is the outcome of work in the class room. Field work should receive as much importance as class work because it is ultimately knowledge in the former aspect that the future citizen requires. Angles should be measure not only on paper but also with the help the clinometers in respect of elevation and depression.

Drawing to scale should be taught with respect to real date collected as a result of field-work and it should form the basis of indirect measurement and calculation. The knowledge gained in classroom should be applied for laying out play field and flower-beds. It should be used in pattern drawing and design-making.

During this stage the learning process should include, besides estimating, drawing and measuring, activities like paper-folding and paper cutting-under such discoverable facts come those relating to area of triangle, area of parallelogram, area of rhombus, area of trapezium, congruency of triangles, symmetry of figures and a good number of geometrical theorems and riders.

13.8 CORRELATION OF MATHEMATICS

Mathematics: with its special features and looks has wider applications in daily life and other fields of study. This facilitates the correlation of mathematics with other fields and disciplines. A teacher of mathematics with a knowledge of the correlation of mathematics can enhance the effectiveness of his/her instructional process by relating it to other areas of interest.

Types of Correlation

Correlation of mathematics can be viewed in the following ways.

- Correlation of mathematics with life

- Correlation of different branches of mathematics
- Correlation between the different topics in the same branch
- Correlation of mathematics with other subjects

13.8.1 Correlation of Mathematics with Life

Mathematics is one subject which has extensive application in our day to day life situations. It has an important bearing on various aspects of life. Mathematics is an indispensable tool of precision in measure involving quantity and time. A fundamental knowledge of basic mathematics concepts is valuable even for a layman. In his day-to-day life when he calculates his wages, plans his expenses, and estimates his balances, he is making use of a lot of simple mathematics. For an intellectual understanding of contemporary literature and to lead a successful life in society, a knowledge of mathematical language, and symbols and their manipulations are essential. Even for a student who discontinues his education, after his primary or secondary education, the fundamental knowledge in mathematics helps him in taking up a good number of vocations such as tailoring, carpentry etc. A knowledge of elementary mathematical concepts such as interest rate, banking, percentage, discount, ratio and proportion, variation (to list a few) is very essential to lead a fruitful life in the society. While teaching mathematics, the teacher of mathematics should cite examples from mathematics to stress upon the practical applications of mathematics. This will make the learning of mathematics interesting and meaningful.

13.8.2 Correlation of Different Branches of Mathematics

The different branches of mathematics such as arithmetic, algebra, geometry, trigonometry should not be taught in watertight compartments. The Indian Education Commission (1964-66) has recommended an integrated approach relating the different branches of mathematics. The set language and concept of function may be used to integrate arithmetic, algebra, geometry and analysis. Though trigonometry is a part of geometry, while dealing with trigonometric functions, it is a part of analysis. The concept of mathematical structures will go a long way in relating the different branches of mathematics to one another. Difficult arithmetic problems can be solved using algebraic equations. Pythagorean theorem and theorems about similar triangles could form the basis for coordinate geometry. The concept of locus provides a correlating link between plane and solid geometry. Geometry could serve as the basis in arithmetic for the topics in mensuration. For example, $\sqrt{2}$ is the hypotenuse of an isosceles right angled triangle whose sides measure 1 unit each. Similar triangles give meaning to ratio and proportion. Similarly, arithmetic should form the basis for algebraic processes. For example, what number should be added to 32 to get 25? This can be solved by forming the algebraic equation $32 + x = 25$. Any linear equation can be represented by straight line. Algebraic terms can

be viewed as certain geometric concepts. For examples 'x' - a variable as a line segment of length x units, x^2 - area of a square of side x units, x^3 - volume of a cube of side x units and so on.

An illustration to show how algebra, geometry and arithmetic can be correlated to one another is given below.

Algebra: Expansion of the identity $(a + b)^2$.

This can be expanded by multiplying (a + b) (a + b) and collecting the like terms and adding them together i.e., $(a + b)^2 = a^2 + b^2 + 2ab$.

Geometry: $(a + b)^2 = a^2 + b^2 + 2ab$.

$(a + b)^2$ represents the area of a square of side (a + b) units which can be further divided into two squares of sides a and b respectively and two rectangles of sides a, b.

Arithmetic:

The same identity can be used to compute 101^2 or 1003^2 and so an $(101)^2 = (100+1)^2 = 100^2 + 2 \times 100 \times 1 + 1$

13.8.3 Correlation of Topics in the Same Branch of Mathematics

If we take any branch of mathematics, the topics in the same branch of mathematics should be correlated to each other. For example, in algebra the topic 'polynomials' is related to equation. Similarly the knowledge of solutions of linear equations is a prerequisite for solving simultaneous equations. Factorisation of quadratic expressions helps in solving quadratic equations. In geometry, the knowledge of area of triangles leads to the derivation of the formulae for the area of a quadrilateral, rectangle, square, parallelogram, rhombus, trapezium, regular hexagon and so on. Similarly, in arithmetic, without the knowledge of percentage, one cannot understand the concepts of discount, interest rate etc. Unless the student is familiar with the concepts of simple interest, he cannot proceed with the computation of compound interest, recurring deposit and so on. Thus in any branch of mathematics, topics are sequential in nature and they are related to each other and dependent on one another.

13.8.4. Correlation of Mathematics with Other Subjects

With Physical Sciences

The correlation of mathematics with sciences is well brought out in the following quotations.

"Mathematics is the gateway and key to all sciences" *Bacon.*

"Mathematics is the indispensable instrument of all physical researches" - *Kant.*

"All scientific education which does not commence with mathematics, is of necessity, defective at its foundation"- *Comte.*

"We cannot overstress the importance of mathematics in relation to science education and research. This has always been so, but at no time has the significance of mathematics been greater than today" - *Kothari Commission.*

Mathematics gives a workable symbolism for the brief and precise expression of ideas to all sciences. All laws and principles in physics are expressed as equations and formulae using mathematical language and symbols. A few examples are given below.

$V = u + at$, $V^2 = u^2 + 2as$ $S = ut + \frac{1}{2} at^2$ (Laws of Motion)

$F = ma$ (Newton's Second Law)

$D = \frac{m}{v}$ (Density Quantified)

$PV = RT$ (Boyle's Law)

In order to understand the above mentioned laws and principles, a student needs a thorough understanding of equations, direct variation, inverse variation and so on. A teacher of mathematics, while teaching equations and variations can make use of examples from physics. Such an approach will make mathematics learning more significant. The quantitative measurements and their manipulations are very essential for understanding physics. For example, a mere knowledge of facts like "steam can generate power" or 'light changes its direction in going from a rarer to a denser medium" is meaningless till they are quantified and applied to practical situations. This is possible only through applying mathematics.

Mathematics is an indispensable tool for a better understanding of chemistry. In the words of J.W. Mellor, "It is almost impossible to follow the later developments of physical or general chemistry without a working knowledge of higher mathematics". Mathematics gives form, shape and definiteness to the properties of matter. All chemical combinations are governed by certain mathematical laws. In chemical compounds, the constituent elements are combined in a definite ratio. In chemical reactions, the chemical equations are balanced by balancing the number of atoms on either side of the equation. The amount of heat generated or required in various chemical reactions are not empirical but mathematically based. The structure of an atom, the atomic weight, and the valence of an element are all mathematically based. Thus mathematics helps to make physical sciences more interesting and practical, and the opportunities for correlation are endless.

With Biology

John Perry has correctly estimated the value of mathematics in the study of Natural Sciences by saying " In these days, all men ought to study natural sciences, such a study is practically impossible without the knowledge of higher mathematical methods". The Indian Education Commission has also emphasised the importance of mathematics in the development of biological sciences in the following lines. "Apart from its (mathematics) role in the growth of physical sciences it is now playing an increasingly important part in the development of biological sciences" Thus biology and medicine which seemed to have practically no use for advanced mathematics at the beginning of the last century, are benefiting immensely from the intervention of sophisticated mathematical tools. The mathematical procedures of biometry are utilised for generalisations arrived from observed data. Statistical methods and techniques are widely used for analysing complex problems relating to physiology, genetics, heredity, metabolism and so on. Bacterial growth is expressed as an exponential function. It is now tnought that the area of topology can be used in describing the surface of the living cell. A new kind of algebra to represent the thinking process is being sought by neurophysiologists. Instances of correlation of mathematics can be continued indefinitely. Of late Biomathematics has been emerging as an important field of study for Biologists.

With Engineering

Most engineering disciplines interact with mathematics in a similar way. The level of sophistication of the mathematics used in dealing with engineering problems has grown by leaps and bounds with its ability to handle more and more complex problems. That is why those who opt for engineering course should have learned mathematics for a better understanding of the subject. Mathematics forms the basis for all engineering courses. A great deal of mathematics used in engineering is in the area of differential equations, an offshoot of the calculus. Probability theory is another area with profound applications to engineering problems. Till recently, algebraic geometry, a relatively 'pure' domain, has been used with considerable success in handling engineering problems.

With Social Sciences

Economics, Psychology and many other subjects are mathematically-oriented. In economics, mathematical language and methods are frequently used to interpret social phenomena, generalise laws governing economic policy and predict economic growth. The theory of probability is applied to many of the economic issues to draw inferences and predict outcomes. Statistical analysis is the only scientific method to deal with social and psychological phenomena. In the words

of Herbert, "It is not only possible, but necessary that mathematics be applied to psychology".

Logic and philosophy are regarded as sciences only in so far as they are mathematical. Logic is the scientific study of the conditions of accurate thinking and drawing valid inferences. Mathematics is the only field of knowledge in which the logical laws can be applied and results verified without any personal bias or prejudice Dr. Alembert says, "Geometry is a practical logic, because in it rules of reasoning are applied in the most simple and sensible manner" while Pascal has gone to the extent of saying "Logic has borrowed the rules of geometry". Philosophy is defined as the science which investigates the ultimate reality of things and that there is no better tool than mathematics to assess the exactness of the realities. According to A.N. Whitehead "philosophers when they have possessed a thorough knowledge of mathematics, have been among those who have enriched the science with some of its best ideas". This implies that a good philosopher should be a good mathematician too, who is well versed with mathematical ideas and techniques to evaluate the validity of the truth and reality.

History and Geography are also correlated to mathematics. The history of mathematicians and their contributions provide useful background for the teaching of mathematics and history. Mathematics helps in calculating the dates and days of various historical events and in predicting the future based on the past events. Geography also makes extensive use of mathematics. Mathematics helps in drawing the maps to the scale and locating places and estimating the distance between different places. Mathematics provides the basis for computing the longitude and latitude, in predicting the weather conditions and climatic changes, planning the transportation system and so on. Problems from history or geography can be a fruitful exercise in mathematics.

With Language and Literature

Language is the vehicle for communication and mathematics cannot be learned without the use of language. Though mathematics has its own language of symbols, they become more clear, meaningful and operational only when they are clearly explained in clear simple language. Certain mathematical concepts, especially abstract concepts can be learned only in the form of verbal statements known as definitions. Such definitions become specific and concise when stated in simple, clear language. In turn, mathematics helps the students to learn the language with clarity and exactness. The logical thinking and reasoning that the students develop through the study of mathematics can be applied to learn the language, especially in the usage of grammar and organising ideas logically and precisely.

With Art and Architecture

Mathematics helps in appreciating the beauty in art and architecture.

A person with a mathematical background can perceive qualities such as symmetry, pattern, proportion, balance and harmony in creations of art and architecture. Shaw states, "Mathematics is engaged, in fact, in the profound study of art and the expression of beauty" Therefore, mathematics itself is a piece of art, a study of harmony and of symmetry. Though there is a large chunk of mathematics labelled as applied mathematics, there is a large body of mathematical knowledge whose creation was driven by purely aesthetic considerations. The artists, in turn, weave their knowledge of form, symmetiy and proportion into their creations to make it beautiful.There is beauty, symmetry, pattern and rhythm in mathematics and one could derive happiness and satisfaction from mathematics like any other piece of art. Its beauty, elegance, and precision may appear sometimes to be cold or inhuman, and yet even very young students can gain great pleasure from working and solving problems within its domain

Conclusion

Mathematics may be thought of as a highly disciplined mode of thinking. The teachers should help students to appreciate the structure and pattern which underlie mechanical and computational skills. Many situations can be broken down by analysing them into interrelated constituent problems which can be explored by well-known mathematical techniques. Wherever there is structure, relationship, regularity, systematic variation, there is mathematics. To recognise this, one needs some knowledge of mathematical skills and formulae, but above all one needs imagination, appreciation of order, structure and pattern, combined with a flexible, roving interest to live in the changing, challenging and exciting world around us.

Check Your Progress

Notes : a) Write your answers in the space given below

b) Compare your answer with the one given at the end of the unit

4. Why do we motivate the students?

……

5. Write a short note on Rationalisation

……

6. How does concretization help for better understanding of the Mathematical concepts?

……

7. How does Mathematics Correlate with Life?

……

13.9 INDIVIDUALISATION OF INSTRUCTION

In a traditional classroom, instruction is teacher-centred and group-paced. It caters to the needs of 'average' students and does not make allowances for the vast individual differences found in the classroom. Differences among students are vast and varied. Students differ in their interests, aptitude, attitudes, intellectual abilities and in a number of other aspects such as pace of learning, learning style, cognitive style and so on. If instruction is to be effective, it must reach individual students and promote their learning as individuals. Individualised instruction is one alternative to traditional approach.

Individualised instruction has been one of the recent important innovations in the field of education. Successful efforts in U.S.A., U.K. and other countries to individualise instruction have prompted educationists to introduce the concept of individualised instructions in our country. The use of technology has made individualisation of instruction more effective and popular.

Meaning of Individualised Instruction

- Individualisation means tailoring of instruction to the particular needs and abilities of the learner.
- In individualisation, the teacher works on a personal one to one basis with each student
- Individualisation envisages the learner to work alone at his own pace.
- In individualised instruction the following factors are adapted to the needs of each individual student; pace, medium of presentation, study style, contents and evaluation technique.

It may be summarised that an instruction is individualised when it provides each learner a major role in the selection of objectives, materials, procedure and time.

Definition of Individualised Instruction

Individualised Instruction is an arrangement that makes it possible for each student to be engaged in learning those things that are most appropriate for him as an individual. International Dictionary of Education defines individualised instruction as follows.

"Individualised instruction is that tailored to the needs of individual students and situations characterised by such features as clear objectives or outcome specifications; detailed-repertoire assessment of the student; active responding or frequent monitoring of student responses; immediate and frequent feedback; use of successive approximation; self- paced learning; mastery learning".

Individualised instruction requires to adjust the instruction to cater to the specific needs and circumstances of each individual learner. It is the accommodation of instruction to the abilities, goals, and pace of learning of each student. In this method, the student assumes the responsibility of learning. The student actively participates in learning and accomplishes his learning task by the frequent and immediate feedback from the teacher.

Characteristics of Individualised Instruction

The general characteristics of individualised instruction have been listed below.

- Defining specific objectives: Terminal behaviour to be achieved is to be specified in operational terms.
- Repertoire assessment: The entry behaviour is to be measured prior to the learner's entry into a given instructional sequence.
- Individual prescription: The instruction is planned according to the needs, capabilities and interest of the individual.
- Active responding: The learner is required to respond actively.
- Reinforcement: Reward or reinforcement follows precisely defined behaviour of the learner.
- Immediate feedback: The learner is immediately informed on the outcome of his performance on a task.
- Frequent feedback - Frequent feedback is arranged to maximise opportunities for the learner to assess the adequacy of his performance.
- Successive approximation: The learner is led to terminal behaviour gradually step-by- step.
- Self-pacing: The learner learns at his own pace.
- Mastery criterion: The learner is required to exhibit a high level performance of the outcome specified as a condition for progress to the next step.
- Presentation of the content in small steps: The material is broken into learning points and are presented in small steps.

Advantages of Individualised Instruction

- Individualised instruction caters to individual differences. Students can be placed in different but suitable programmes based on their unique needs and abilities.

- Individualised instruction allows the learner to proceed at his own pace based on his need and level of achievement.
- Individualised instruction helps in the maximum development of the potentialities of the learner.
- The feedback provided at regular intervals helps the students to gain mastery over the content under study.
- This method is suitable, for remedial purposes as the teacher can identify the weak areas from the record of each easily accessible to the teacher

13.10 HOME ASSIGNMENT

School time is not sufficient to exhaust everything that a student has to learn in mathematics. Homework is to be given to supplement the school work and to provide for application and practice necessary for fixation of ideas, concepts, rules and formulae. Moreover, homework helps the students to recapture what has been done in the class that day. It provides the link between each day's work and next day's work.

Purposes of Homework

The chief purposes of homework are

a. To provide drill on operations whose theory is understood.

b. To fix in memory rules and formulae that need to be memorized.

c. To inculcate the habit of regular and systematic work.

d. To provide opportunity for quiet thinking and independent work.

In spite of the several advantages of homework, many educationists have vehemently criticized the assigning of homework to school students. The following are the criticisms leveled against homework:

- The students are forced to carry the school atmosphere to the house.
- The burden of the homework might make the student hate the subject, the teacher or even the school.
- Overburden of the homework affects the child both physically and emotionally.
- It deprives the students of their leisure time.
- All students may not have the same congenial atmosphere at home conductive for doing homework.

- It is educationally unsound to overload the students with homework
- Constant anxiety of some work adversely affects the child's health and retards his academic progress.

Merits of Homework

The importance and utility of homework cannot be overlooked. When homework is made enjoyable, useful and attractive, it is of very great valuable to the students. The following are the advantages of homework:

a. It encourages independent work and self-responsibility

b. It develops the habit of regular and systematic work.

c. It helps in clarifying the doubts of the students.

d. It supplements classroom work.

e. It provides a link for parent-teacher co-operation.

f. It forces the students to revise what has been done in the class.

g. It helps in fixation of ideas and principles.

Some Suggestions for Making Homework more Effective

It is very important that homework should be made useful and effective. The most effective homework is that which has the character of completing the class work of the previous day. It is not advisable to assign work unless it has been sufficiently developed in the class to enable even the dull pupil to apply his time to it with success. The pupil should never be made to struggle with new matter without the supervision of the teacher. However, the work assigned may require thinking, solving new problems, provided the students could find some clue to the solution from the problems that had been worked out in the class. The following points may be kept in mind while planning homework.

a. Homework should be interesting and challenging; it should not be boring and burdensome.

b. It should be well graded to suit the individual's capacity and interest

c. It should not be too difficult or too easy

d. Homework should not be given in a particular lesson until it has been clearly understood even by the slow learners.

e. As far as possible, home conditions of the students should be borne in mind while assigning homework.

f. Homework should be made interesting by assigning practical work relating to the unit such as preparing models, collecting data, field work and so on.

g. Homework should never be made as a means of punishment.

h. Homework timetable should be prepared in consultation with other teachers so that it does not overburden the students.

i. Homework should be regularly checked and feedback should be given to the students.

Correction of Assignments

It is not enough that the teacher gives intelligently planned homework. Correction of the assignment is equally important. The teacher has to check the assignments for accuracy and understanding and feedback should be given to the students. This will help the students to gauge their own understanding of the subject. The common misconceptions may be pointed out to the whole class for clarification and individual mistakes to the respective individuals. Through understanding of the content results in minimum errors. The teacher should insist on neat and systematic work resulting in minimum errors. A progress chart can be maintained to monitor and keep track of the student's work. The teacher may use a grading system to assign relative positions to the student on the basis of their homework.

Check Your Progress

Notes : a) Write your answers in the space given below

b) Compare your answer with the one given at the end of the unit

8. What is Individualised Instruction?

……

……

9. Mention any four advantages of Individualised Instruction

……

……

10. Write a short on Home Assignment.

……

……

13.11 LET US SUM UP

Mathematical skills need to be perfected and maintained through systematic drill. Concepts and relationships must be reviewed and applied at frequently recurring intervals. This unit discusses the importance of drill and review which are the means to strengthen the knowledge already acquired. Drill and exercises have a valuable place in the teaching of mathematics.

Motivation will enable the students, to learn diligently and purposively.Effectiveness of learning is subject to two factors – understanding and responding. Suitable means of motivation should be employed to ensure that the teaching is impressive.

In this unit, rationalization and concretisation have been explained to help in better understanding of the Mathematical concepts.A teacher of mathematics with a knowledge of the correlation of mathematics can enhance the effectiveness of his/her instructional process by relating it to other areas of interest. In this unit, the emphasis has been laid on the correlation of mathematics with other subjects.

The importance and utility of homework has been explained so that the teacher can be able to make the homework more useful that will help the students to recapture what has been done in the class that day. When homework is made enjoyable, useful and attractive, it is of very great valuable to the students.

13.12 UNIT-END ACTIVITIES

1) Explain the different ways by which drill work can be conducted in mathematics.
2) How does review differ from drill?
3) What do you understand by review? What is its place in the teaching of mathematics? How will you make review more effective?
4) How is homework indispensable in the teaching of mathematics? Give some hints to make it more effective as a technique?
5) How is mathematics correlated with other school subjects? Illustrate.

13.13 ANSWERS TO CHECK YOUR PROGRESS

1. Demerits of Drill
    - Mechanical reproduction of ideas.
    - Rote memorization of principles and formulae
    - Monotonous and routine work and
    - Learning without clear understanding.
2. Review is sometimes identified with drill as both are characterized by repetition and both aim at fixations of ideas. However, a review is different from a drill as it aims not only at fixation of ideas but also viewing the material differently and in a wider perspective for better understanding of the material. It is the thoughtful organisation of the important details of the content into a coherent whole in order that relationship of the various

elements to each other and to whole unit may be understood. Review emphasises thought and meaning rather than habit formation.

3. Suggestions for effective review
   a. The emphasis in review should be on thought and meaning rather than habit formation.
   b. It should not involve mere repetition of facts, but meaningful reorganization of the subject matter into a coherent whole.
   c. It should lead to integration of facts and ideas.
   d. It should result in new learning rather than fixation of already learned ideas and concepts.
   e. It should reveal the relationship among the various elements of the subject matter.
   f. It can be integrated with daily classroom teaching or can be given as a specialised assignment.
   g. It should provide strength and coherence to the entire structure of knowledge through continual re-association of the components.
   h. Definite instructions as to how to review the lesson should be provided to the students.
4. One of the important problems of mathematics teaching is that of creating and maintaining interest in the subject. This task is called, motivation. Motivation should be secured for two distinct purposes namely, following classroom teaching and taking an abiding interest in mathematics. Motivation will enable the students, to learn diligently and purposively. The teacher has to take note of the various interests and experiences of the students. Thus the problem of motivation reduces itself to that of creating interest as well as sustaining it throughout the learning process.
5. The study of Mathematics develops reasoning power. Rationalization helps problem solving. Rationalization of Mathematical processes, working rules and formulae is a must for effective learning. If we develop the rational first and then arrive at the working rule. We are employing Mathematics primarily as a mode of thinking and secondarily as a tool. Generally rationalized teaching is preferable to mechanical. Rationalization helps the pupil to satisfy that there is justification for every step in his procedure and to recall the missing links in his reasoning by resorting to systematic analysis of the "if – then" pattern, retracing the steps for the purpose.

6. Concretisation helps betterunderstanding of the Mathematical concepts. Audio-visual aids help very much in concretising abstract concepts. The teacher in order to concretise abstract principles can relate them to the direct experience of the students. Concretisation is much more needed in the lower classes than in the higher classes. Practical work helps to concretise same abstract ideas.

7. Mathematics is one subject which has extensive application in our day to day life situations. It has an important bearing on various aspects of life. Mathematics is an indispensable tool of precision in measure involving quantity and time. A fundamental knowledge of basic mathematics concepts is valuable even for a layman. In his day-to-day life when he calculates his wages, plans his expenses, and estimates his balances, he is making use of a lot of simple mathematics.

8. Individualised Instruction is an arrangement that makes it possible for each student to be engaged in learning those things that are most appropriate for him as an individual. International Dictionary of Education defines individualised instruction as "Individualised instruction is that tailored to the needs of individual students and situations characterised by such features as clear objectives or outcome specifications; detailed-repertoire assessment of the student; active responding or frequent monitoring of student responses; immediate and frequent feedback; use of successive approximation; self- paced learning; mastery learning".

9. Advantages of Individualised Instruction

    - Individualised instruction caters to individual differences. Students can be placed in different but suitable programmes based on their unique needs and abilities.
    - Individualised instruction allows the learner to proceed at his own pace based on his need and level of achievement.
    - Individualised instruction helps in the maximum development of the potentialities of the learner.
    - The feedback provided at regular intervals helps the students to gain mastery over the content under study.

10. School time is not sufficient to exhaust everything that a student has to learn in mathematics. Homework is to be given to supplement the school work and to provide for application and practice necessary for fixation of ideas, concepts, rules and formulae. Moreover, homework helps the students to recapture what has been done in the class that day. It provides the link

between each day's work and next day's work. It is very important that homework should be made useful and effective. The most effective homework is that which has the character of completing the class work of the previous day. It is not advisable to assign work unless it has been sufficiently developed in the class to enable even the dull pupil to apply his time to it with success. The pupil should never be made to struggle with new matter without the supervision of the teacher.

13.14 SUGGESTED READINGS

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UNIT – XIV
TEACHERS AND PROFESSIONAL DEVELOPMENT

Structure

14.1 INTRODUCTION

As in the case of other teachers, many things are expected from the teacher of mathematics. His obligations not only are confined to the classroom but extend in many other directions also.

The beginning teacher will need to spend most of his time in improving his knowledge of his field and techniques of teaching and becoming familiar with the traditions and administrative policies of the school.

The role of mathematics teacher is essential for highly effective mathematics instruction in the development of the learners' mathematical knowledge and mathematical ability.

This unit describes the characteristics and role of the good mathematics teacher and the professional growth of mathematics teacher.

This unit equips the student teacher with adequate information about AMTI and NCTM and also how to play a role in mathematical Olympiads and the importance of contribution of research articles to journals and magazines.

14.2 OBJECTIVES

At the end of the unit, the student-teachers will be able to

- Aware the characteristics and roles of mathematics teacher.
- Gain knowledge of several mathematical organization for professional advancement.
- Aware of the value of mathematics Olympiads.
- Understand the importance of professional growth for a mathematics teacher.
- Know about personal, academic, emotional, intellectual qualities which is needed for a good mathematics teacher.
- Realize the administrative and departmental duties of a mathematics teacher.

14.3 MATHEMATICS TEACHER

Everything in education is planned and set for the benefit of students. If students are not there, educational institutions will not be there. Students are entering the academic environment not only to gain bookish knowledge but also to gain varied learning experiences. Goal of any educational institution is to modify the behaviour of students in positive way and bring about total personality development of learner.

- Who brings all this change?
- Who imparts the information to the child?
- Who makes a slow learner, an average child?
- Who stimulates students thought make their creative mind in action?

- Who brings students to ground and make them do different activities and participate in different games?

That person or individual is teacher. The teacher is more like a gardener, who tends each plant, gives water and sees the plant food is available in soil so that plant may take its own nourishment.

His obligations not only are confined to the classroom, but extend to many other directions too, but the first and foremost obligation of teacher is to teach his subject effectively.

Dr. S. Radhakrishnan emphasizes the role of teacher "the teacher's place in the society is of vital importance. He acts as a pivot for transmission of intellectual traditions and technical skill from generation to generation, and helps to keep the lamp of civilization burning. He not only guides the individual, but also so to say, the destiny of nation. Teachers have therefore to realize their special responsibility to the society. On the other hand, it is incumbent on the society to pay due regard to the teaching profession and to ensure that the teacher is kept above want and gives the status which will command respect from his students".

14.3.1 Role of a Teacher

Pre-service education of prospective teachers places a heavy emphasis in instructional skills of teacher. However, in reality, they are also required to discharge many other responsibilities.

According to Armstrong and Savage, (1994) following is comprehensive list of a teacher's role:

- Instructional role.
- Counseling role.
- Management role.
- Professional development role.
- Curriculum development role.
- Public relations role.
- Non-school role.

14.3.2 Role of a Mathematics Teacher

"To teach is to transform by informing to develop a zest for life-long learning, to help pupils become architects of an exciting challenging future."

\- Edgar Dale.

There is an old saying that 'Teachers are 'born' and not 'made' but current society is so terrific and horrible (if said). We can say that teachers are 'made' and 'not born', if 'born' number is restricted.

Teaching mathematics is a task, which, if sincerely undertaken will challenge the best efforts of the teacher. No teacher can do a thoroughly a good job of teaching unless he is willing to make a careful analysis of his job. Mathematics is a subject which contains theory, practical, games, creativity and practice. Role of mathematics is very profound, because if reality is seen, 80% of students HATE mathematics subject and this hatred is only due to attitude shown by Mathematics teacher.

Important Functions are

(1) Development of love for mathematics: is the pressing function of mathematics teacher, so as to make mathematics education without tears.

(2) Impart mathematics education in scientific manner: plan your teaching/learning process according to 3 A's – (1) Age, (2) Ability and (3) Aptitude of your classroom group of students.

(3) Mathematics teacher should see that boredom never knocks the door of her lecture-hall. Create interest in topic by using different approaches like asking Heuristic questions, assigning activity through Project method, using play-way approach and bringing different types of innovative techniques in classroom.

(4) Inspire your students and dismiss fear from their minds about wrong belief that 'Mathematics is difficult subject' to pick up.

(5) Bring good rapport with students, to remove their difficulties and have periodic contacts with parents, to brief student's achievements in subject.

(6) Organize Mathematical club and plan out different activities under the club. Ensure that maximum students and teachers participate in activities.

(7) Teachers of primary section teaching the subject should take care that they make basics of mathematics very strong in students. If basic foundation is strong, complicated and abstract problems will appear zero to students.

(8) Motivate students to read different books in mathematics.

(9) Appreciate the different values of mathematics like aesthetic, culture, disciplinary, practical etc., in front of the class.

(10) Mathematics teacher should himself/herself have positive attitude towards the subject and always try for academic up-gradation.

This plus the administrative functions are main functions of mathematics teacher.

14.4 QUALITIES OF MATHEMATICS TEACHER

Teacher

As American Commission on teacher education has analyzed – "The quality of a nation depends upon the quality of its citizens, the quality of its citizen depends not exclusively, but in critical measure upon the quality of their education. The quality of their education depends more than any other single factor, upon the quality of their teacher."

According the Kothari Commission, (1966) – "Of all the different factors of education and its contribution to national development, the quality of teachers are undoubtedly the most significant".

The job of a mathematics teacher is very delicate, as he is teaching a subject which is called "an exact Science". Therefore a mathematics teacher should not simply be "What he SAYS AND DOES" but largely by "What he IS".

14.4.1 Professional and Academic Qualities

(1) In depth knowledge of mathematics subjects: This will lead to mastery in content, building strong confidence in mathematics teacher.

(2) Updated knowledge of pedagogy: The different methodologies, strategies, maxims, knowledge and also knowledge of choosing an appropriate methodology suitable to content matter and student ability, should be one of qualities of mathematics teacher.

(3) Knowledge of child psychology: Teacher should have full knowledge of learning, growth, development of children, interest, and aptitude, problem of adolescence, laws and theories of learning.

(4) Love for mathematics: If teacher has love and interest for mathematics, he would be able to create love and interest in mathematics in his students.

(5) A favorable attitude towards teaching profession: Mathematics teacher should love teaching and should have commitment to it, should appreciate institution, his work place and have love for children.

(6) Knowledge of goals of institution: Mathematics teacher apart from knowing and chalking out his/her objectives should also know goals of his institution. He/she should maintain respect for his institution and dignity.

(7) Knowledge of instructional materials: Textbook, Reference book, Audio visual aids, programmed instructional booklet, CAI packages, self-learning materials, Audio-Video cassettes, slides, should be skillfully used by mathematics teacher.

(8) Communication skill: Teacher should have ability to listen carefully, attentively to others, be able to communicate her ideas, clarity of voice, preciseness, and clarity of speech, well-modulatedvoice, ability to pause at appropriate time and adequate facial expressions, body gestures.

(9) Adaptation: Teacher should adapt oneself to any situation and be able to impart information, under any condition.

(10) Classroom skills: Teacher should have various micro-teaching skills, C.B.W. Skills, planning and organizing lesson to win confidence of students, to give feedback and ability and to use rewards and punishment judiciously.

(11) Withholding negative expectation of student's performance: Teacher herself should have positive attitude towards life and subjects and express appropriate positive feelings and expectations to students.

(12) Going to class well-equipped: Teacher herself should plan and organize her lessons, set-indication, developing units, closure, use of Audio Visual Aids etc. Adequately well in advance, so that she can impart the contents adequately and effectively.

(13) Ability to admit her weak points: Mathematics teacher should admit lack of knowledge, information and have appropriate will and motivation to rectify such weaknesses by additional professional reading.

(14) Knowledge of application of mathematics: Mathematics is applicable in every walk of our life practical cultural, social, moral,discipline and aesthetic. Teachers should know how to impart all those values to students.

(15) Knowledge of analytical skill: Mathematics should be taught minutely, giving every detail of the content. Teacher should be able to break the problem and analyse it.

(16) Organizing different activities in mathematics subject: A mathematics teacher should organize different activities,

under the banner of Mathematics club, teacher should have capacity to undertake visits like to Nehru Science Centre, exhibitions conducted by different institutions, on different working models, enthuse students by exciting their thinking by organising quizzes, riddles, mathematical jokes, competitions, debate, discussions, extempore competition, talk by experts, Audio Video Aids, Collage competition the list is endless.

(17) Adequate general knowledge: Mathematics teacher should be an all-rounder. His scholarship will be incomplete unless he has his hands on the pulse of his surroundings and even on near and far – away things. He should be jack of many trades and master of mathematics.

(18) Sense of humor: Even this is an academic and professional quality of teacher of mathematics. In mathematics class, generally the situation is tense one. The solution to problems are not achieved so easily. Proofs are not obtained for a particular theorem. Some concepts are abstract and they bounce on head of students. Some formulas are complicated. Humour, at such times, serves as a tonic in difficult situation. It relieves tension and motivates students and refreshes them to learn mathematics.

(19) Having urge for professional up-gradation: Mathematics teacher should always be ready tom participate in faculty meetings, mathematical workshops, seminars, orientation, classes, exhibitions, projects, fairs, educational visits, attending lectures of experts, reading books, undertaking research works.

(20) Presence of mind: This is basic requirement of any teacher. To solve problems of students, subjects and classroom situation, this quality should be there in all teachers.

(21) Knowledge about different branches of mathematics: Teacher should have knowledge of Geometry, statistics etc.

14.4.2 Personal Qualifications

(1) Honest
(2) Punctual
(3) Orderliness
(4) Intellectual Honesty
(5) Impartiality
(6) Objectivity

(7) Democratic attitude
(8) Ability to lead others
(9) Open-mindedness
(10) Initiative
(11) Motivation
(12) Perseverance
(13) Hard work
(14) Integrity
(15) Moral Values
(16) Positive mental health
(17) Adequate self-concept
(18) Neat and clean personal appearance
(19) Innovative-proneness
(20) Authentic
(21) Humble student of mathematics
(22) Energetic
(23) Sense of Justice
(24) Ability to handle conflicts constructively
(25) Qualities of leadership
(26) Patience
(27) Progressive and dynamic outlook
(28) Habit of self-study
(29) Good relation with teachers and students
(30) Logical thinking
(31) Firmness

14.4.3 Intellectual Qualification

(1) Common sense.
(2) Good memory
(3) Sharp
(4) To Undertake new project
(5) To use innovative technique in classroom situations
(6) Broad-based knowledge
(7) Computational skill, drawing and sketching skill.

14.4.4 Emotional Qualities

(1) Emotional stable.
(2) Sense of humour
(3) Artistic touch
(4) Optimistic
(5) Assertiveness

(6) To provide guidance and remedial teaching to special children.

14.4.5 Physical Health

(1) Healthy

(2) Vitality

(3) Good Physique

14.4.6 Professional Qualifications of Mathematics Teacher

(1) Graduate in Mathematics subject, from Recognized University.

(2) B.Ed. or equivalent degree from Recognized University.

(3) Diploma in Education from recognized University.

(4) Attending and participating in seminar, workshop organized by reputed Institution as an in-service course.

(5) Doing professional research work.

(6) Written professional articles or books for publication.

(7) Be member of Mathematics Organization at District, state or National level.

(a) Administrative Duties:

(i) Managing class register

(ii) Collecting fees

(iii) Collecting Fee for Maths Club.

(iv) Collecting fees for educational tour.

(v) Conduct examination

(vi) Prepare result

(vii) Sending Progress report to students

(viii) Offering guidance and Co-operation to students, parents, local community, and managing body of Education.

(ix) Helping students to select and execute projects.

(x) Supervising some other important activities of the school.

(b) Departmental Duties:

(i) Preparing Tests in mathematics

(ii) Evaluate the Tests

(iii) Selecting good textbooks, periodicals, magazines, journals for library

(iv) Organise course content

(v) Selecting appropriate workbook, teacher's guide

(vi) Participate in different departmental meetings and discussions

(vii) Organize Mathematics club and planning out different actives under it.

(viii) Selecting appropriate industrial material like models of prisms, rectangular pyramids, cuboids, sum of measure of sum of measure of triangle is 180° degree, Pythagoras theorem etc.

(ix) Diagnosis and remedial teaching of special students.

(x) Arranging supervised study.

14.5 PROFESSIONAL GROWTH OF MATHEMATICS TEACHER

B.A., /B.Sc., B.Ed., or M.A. /M.Sc., B.Ed. teacher should not feel contended with his qualifications. He should always strive for professional growth. Professional growth is a career long obligation. Even though a teacher has all the qualities of a successful and creative teacher, it should be clear that all the qualities cannot be imbibed at once, but to be qualitative, he has to strive hard, to pick up most of the qualities in various doses at appropriate occasions. But it should be sooner the better.

Different Activities for Professional Growth

(1) Acquiring formal degrees.

(2) Participating in faculty meetings.

(3) Organising and planning out different activities under mathematical club.

(4) Participating in mathematics workshop/Seminars.

(5) Studying latest mathematics journals, books, publications.

(6) Availing all facilities on in-service training program.

(7) Professional research work should be carried out.

(8) Write professional article or books for publication.

(9) Participate in different discussions of professional organization.

(10) Participate in action-research project, crash and innovative program, attended through his personal

approach in varied in NCERT sponsored, local i.e. SIE arranged and other programs.

(11) Earn grant for his visit to great academic and professional Centers.

(12) Earn training in specialized branches of mathematics.

Once professional growth of teacher starts, it provides teachers with opportunities to extend their expertise. He may get chance to discharge duties as supervisor, to prepare school's budget, distributing material required, coordinating school activities.

A teacher of mathematics should aspire to gain entrance in higher professional challenging institutions by virtue of his work and worth to prove an asset for himself and institution as a very capable Teacher.

Check Your Progress

Notes : a) Write your answers in the space given below

b) Compare your answer with the one given at the end of the unit

1. Mention any five main functions of a mathematics teacher.

   ……

   ……

2. State some academic qualities of a mathematics teacher

   ……

   ……

3. Write a short note on professional growth of a mathematics teacher

   ……

   ……

4. List out some activities for professional growth

   ……

   ……

14.6 SEMINARS

Seminars are simply a group of people coming together for the discussion and learning of specific techniques and topics. Usually there are several keynote speakers within each seminar, and these speakers are usually experts in their own fields, or topics. Attending a seminar has numerous benefits, including improving communication skills, gaining expert knowledge, networking with others and renewing motivation and confidence.

Benefits of attending seminars for a mathematics teacher

- Gaining renewed enthusiasm for teaching

- Looking at teaching with “fresh eyes”
- Learning new information from the presenters
- Meeting new people and sharing experiences
- Brainstorming one’s own ideas and getting immediate feedback
- Availing shifts in understanding teaching
- Becoming more reflective and aware as teachers
- Enhancing the quality of student learning
- Building professional communities

14.7 ASSOCIATION OF MATHEMATICS TEACHERS OF INDIA

The Association of Mathematics Teachers of India or AMTI is an academically oriented body of professionals and students interested in the fields of mathematics and mathematics education.The AMTI's main base is Tamil Nadu, but it has recently been spreading its network in other parts of India, particularly in South India.

The AMTI has been organizing conferences in different parts of the country to meet and deliberate issues of mathematics education, particularly at the school level.

Objectives:

- To assist practising teachers of mathematics at all levels in improving their expertise and professional skills making mathematics interesting and enjoyable.
- To spot out and foster mathematical talent in the young.
- To disseminate new trends in Mathematics Education.
- To offer consultancy services to schools.

Periodicals

- The official Journal of the Association “The Mathematics Teacher (India)” is published quarterly in English and is sent to members free.
- The Journal for students - JUNIOR MATHEMATICIAN - is published tri-annually in English before the commencement of vacation(s) and is supplied only to the subscribers through the respective schools, wherever possible.

Activities

- Conducting National Mathematics Talent Contests. (NMTC)

- Conducting Grand Achievement Test (GAT)
- Arranging Exposure Programmes for talented students.
- Organising Orientation Courses, Seminars and Workshops for teachers including suggestions to equip the mathematics section of their libraries and laboratories.
- Organizing National Conferences in different parts of the country to meet and deliberate on important issues of Mathematics Education.
- Giving Distinguished Mathematics Teacher Award to enterprising and pioneering teachers of Mathematics.

14.8 NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

The National Council of Teachers of Mathematics (NCTM) was founded in 1920. It has grown to be the world's largest organization concerned with mathematics education, having more than 80,000 members across the USA and Canada, and internationally.

The National Council of Teachers of Mathematics presents itself as "the public voice of mathematics education, supporting teachers to ensure equitable mathematics learning of the highest quality for all students through vision, leadership, professional development, and research. NCTM holds annual national and regional conferences for Mathematics teachers and publishes four print journals.

Strategic Priorities of NCTM

- **Access and Equity:** Advance knowledge about, and infuse in every aspect of mathematics education, a culture of equity where everyone has access to and is empowered by the opportunities mathematics affords
- **Advocacy:** Engage in public and political advocacy to focus policymakers and education decision makers on improving learning and teaching mathematics.
- **Curriculum, Instruction, and Assessment:** Provide guidance and resources for developing and implementing mathematics curriculum, instruction, and assessment that are coherent, focused, well-articulated, and consistent with research in the field, and focused on increasing student learning.
- **Professional Development:** Provide professional development to all stakeholders to help ensure all students receive the highest quality mathematics education

- **Research:** Ensure that sound research is integrated into all activities of the Council.
- **Technology:** Promote strategic use of technology to advance mathematical reasoning, sense making, problem solving, and communication.

14.9 MATHEMATICAL OLYMPIAD

The Mathematics Olympiad activity was undertaken by NBHM(*National Board for Higher Mathematics*) from 1986 onwards and is currently run in collaboration with the HomiBhabha Centre for Science Education, Mumbai.

One main purpose of this activity is to support mathematical talent among high school students in the country. NBHM has taken on the responsibility for selecting and training the Indian team for participation in the International Mathematical Olympiad every year. While NBHM coordinates and supports Mathematics Olympiad contests all over the country, regional bodies, mostly voluntary, play an important role at different stages.

For the purpose of administering Mathematics Olympiad contests, the country has been divided into 16 regions. A regional coordinator is responsible for conducting these tests in each region.

General Information about Mathematics Contests

The Mathematics Olympiad Programme leading to participation in the International Mathematical Olympiad consists of the following stages:

Stage 1:

Regional Mathematical Olympiad (RMO) is held in each region normally between September and the first Sunday of December each year. The regional coordinator ensures that at least one centre is provided in each district of the region. All high school students up to class XII are eligible to appear for RMO. RMO is a 3-hour written test containing about 6 to 7 problems. Each regional coordinator has the freedom to prepare his/her own question paper or to obtain the question paper from NBHM. The regions opting for the NBHM question paper hold this contest on the 1st Sunday of December. On the basis of the performance in RMO, a certain number of students from each region are selected to appear for the second stage. Regional coordinators charge nominal fees to meet the expenses for organizing the contests.

Stage 2:

Indian National Mathematical Olympiad (INMO) is held on the first Sunday of February each year at various Centres in different regions. Only students selected on the basis of RMO from different

regions are eligible to appear for INMO. INMO is a 4-hour written test. The question paper is set centrally and is common throughout the country. The top 30-35 performers in INMO receive a certificate of merit.

Stage 3:

International Mathematical Olympiad Training Camp (IMOTC)The INMO certificate awardees are invited to a month long training camp (junior batch) conducted in May-June, each year. In addition, INMO awardees of the previous year that have satisfactorily gone through postal tuition throughout the year are invited again for a second round of training (senior batch).

Stage 4:

International Mathematical Olympiad (IMO)The team selected at the end of the camp, a "leader" and a "deputy leader," represent India at the IMO that is normally held in July in a different member country of IMO each year. The leader and deputy leader are chosen by NBHM from among mathematics teachers/researchers involved in the Mathematics Olympiad activity. IMO consists of two written tests held on two days with a gap of at least one day. Each test is of four-and-a-half-hours duration. Travel to IMO venue and return takes about two weeks. India has been participating in IMO since 1989. Students of the Indian team who receive gold, silver and bronze medals at IMO receive a cash prize of Rs. 5,000/-, Rs. 4,000/- and Rs. 3,000/- respectively, from NBHM during the following year at a formal ceremony at the end of the training camp. The Ministry of Human Resource Development (MHRD) finances international travel of the eight-member Indian delegation connected with international participation. NBHM finances the entire in-country programme and takes care of other expenditure. Students aiming for selection for participation in IMO should note that RMO is the first essential step for the programme. To appear for RMO, students should get in touch with the RMO co-ordinator of their region well in advance, for enrolment and payment of a nominal fee.

Syllabus for Mathematics Olympiads

The syllabus for Mathematics Olympiads (regional, national and international) is pre-degree college mathematics. The areas covered are: number systems, arithmetic of integers, geometry, quadratic equations and expressions, trigonometry, co-ordinate geometry, systems of linear equations, permutations and combinations, factorisation of polynomials, inequalities, elementary combinatorics, probability theory, number theory, infinite series, complex numbers and elementary graph theory. The syllabus does not include calculus and statistics. The typical areas for problems are: number theory, geometry, algebra and combinatorics. The syllabus is in a sense spread over class IX to class XII levels, but the

problems under each topic are of an exceptionally high level in difficulty and sophistication. The difficulty level increases from RMO to INMO to IMO.

The values of Mathematics Olympiads

All teachers should have the awareness of the value of Mathematics Olympiads for their learners. The following are the reasons to why the student's participation in Mathematics Olympiads was important:

- It provides learners an opportunity to improve their problem-solving skills by encouraging deep, lateral, independent and creative thinking.
- It allows learners to enjoy and engage with mathematics in a different context.
- Gifted and talented learners are exposed to challenging problems and given the opportunity to excel.
- During practice sessions, learners are able to discuss different problem-solving strategies and to "think outside the box.
- It helps learners to become confident and develop inquiring minds.

14.10 JOURNALS AND MAGAZINES

The journal provides a forum for the exchange of ideas and experiences which contribute to the improvement of mathematics teaching and learning for students from upper secondary/high school level through to university level. A distinctive feature of the journal is its emphasis on the applications of mathematics and mathematical modelling within the context of mathematics education world-wide. The journal's readership consists of mathematics teachers, students, researchers and those concerned with curriculum development and assessment, indeed anyone concerned about the education of users of mathematics.

Uses:

- Seeks to improve the education of mathematics teachers and develop teaching methods that better enable mathematics students to learn.
- Covers all stages of the professional development of mathematics teachers and teacher-educators.
- Examines institutional, societal, and cultural influences that have impact on teachers' learning and their students' learning.

- Instigates research motivation and reporting findings of the investigation that may be useful, of interest to mathematic teachers and students at different levels.
- Paves way for mathematic teachers and students to develop a broad outlook in assessing what is published and contradicting or supplementing the facts published.

Check Your Progress

Notes : a) Write your answers in the space given below

b) Compare your answer with the one given at the end of the unit

5. What are the objectives of Association of Mathematics Teachers of India (AMTI)?

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6. List strategic priorities of NCTM.

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7. Draw a short note on Mathematics Olympiads

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8. State the values of Mathematics Olympiads

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14.11 LET US SUM UP

This unit has presented important characteristics and roles of mathematics teacher. The job of a mathematics teacher is very delicate, as he is teaching a subject which is called “an exact Science”. Therefore a mathematics teacher should not simply be “What he SAYS AND DOES” but largely by “What he IS”.

Mathematics teachers should not feel contended with his qualifications. He should always strive for professional growth.

Professional growth is a career long obligation. Even though a teacher has all the qualities of a successful and creative teacher, it should be clear that all the qualities cannot be imbibed at once, but to be qualitative, he has to strive hard, to pick up most of the qualities in various doses at appropriate occasions.

Attending a seminar has numerous benefits, including improving communication skills, gaining expert knowledge, networking with others and renewing motivation and confidence.

The Association of Mathematics Teachers of India or AMTI is an academically oriented body of professionals and students interested in the fields of mathematics and mathematics education. The AMTI has been organizing conferences in different parts of the country to meet and deliberate issues of mathematics education, particularly at the school level.

The National Council of Teachers of Mathematics presents itself as "the public voice of mathematics education, supporting teachers to ensure equitable mathematics learning of the highest quality for all students through vision, leadership, professional development, and research. NCTM holds annual national and regional conferences for Mathematics teachers and publishes four print journals.

The Mathematics Olympiad activity was undertaken by NBHM (*National Board for Higher Mathematics*)from 1986 onwards and is currently run in collaboration with the HomiBhabha Centre for Science Education, Mumbai. One main purpose of this activity is to support mathematical talent among high school students in the country. All teachers should have the awareness of the value of Mathematics Olympiads for their learners.

14.12 UNIT – END ACTIVITIES

1) Enlist and explain the qualities and characteristics of teacher of mathematics.
2) Explain AMTI and MCTM.
3) Discuss the role of mathematics teacher in making mathematics learning effective.
4) What are the benefits of attending seminar for a mathematics teacher?
5) Describe Mathematical Olympiad.

14.13 ANSWERS TO CHECK YOUR PROGRESS

1. Important Functions of a mathematics teacher are

- Development of love for mathematics: is the pressing function of mathematics teacher, so as to make mathematics education without tears.
- Impart mathematics education in scientific manner: plan your teaching/learning process according to 3 A's – (1) Age, (2) Ability and (3) Aptitude of your classroom group of students.
- Inspire your students and dismiss fear from their minds about wrong belief that 'Mathematics is difficult subject' to pick up.
- Organize Mathematical club and plan out different activities under the club. Ensure that maximum students and teachers participate in activities.
- Teachers of primary section teaching the subject should take care that they make basics of mathematics very strong in students. If basic foundation is strong, complicated and abstract problems will appear zero to students.

2. Professional and Academic Qualities

- In depth knowledge of mathematics subjects: This will lead to mastery in content, building strong confidence in mathematics teacher.
- Updated knowledge of pedagogy: The different methodologies, strategies, maxims, knowledge and also knowledge of choosing an appropriate methodology suitable to content matter and student ability, should be one of qualities of mathematics teacher.
- Knowledge of child psychology: Teacher should have full knowledge of learning, growth, development of children, interest, and aptitude, problem of adolescence, laws and theories of learning.
- Love for mathematics: If teacher has love and interest for mathematics, he would be able to create love and interest in mathematics in his students.
- A favorable attitude towards teaching profession: Mathematics teacher should love teaching and should have commitment to it, should appreciate institution, his work place and have love for children.

3. B.A., /B.Sc., B.Ed., or M.A. /M.Sc., B.Ed. teacher should not feel contended with his qualifications. He should always strive for professional growth. Professional growth

is a career long obligation. Even though a teacher has all the qualities of a successful and creative teacher, it should be clear that all the qualities cannot be imbibed at once, but to be qualitative, he has to strive hard, to pick up most of the qualities in various doses at appropriate occasions. Once, professional growth of teacher starts, it provides teachers with opportunities to extend their expertise. He may get chance to discharge duties as supervisor, to prepare school's budget, distributing material required, coordinating school activities.A teacher of mathematics should aspire to gain entrance in higher professional challenging institutions by virtue of his work and worth to prove an asset for himself and institution as a very capable Teacher.

4. Different Activities for Professional Growth

    - Acquiring formal degrees.
    - Participating in faculty meetings.
    - Organising and planning out different activities under mathematical club.
    - Participating in mathematics workshop/Seminars.
    - Studying latest mathematics journals, books, publications.
    - Availing all facilities on in-service training program.
    - Professional research work should be carried out.

5. Objectives of Association of Mathematics Teachers of India

    a. To assist practising teachers of mathematics at all levels in improving their expertise and professional skills making mathematics interesting and enjoyable.

    b. To spot out and foster mathematical talent in the young.

    c. To disseminate new trends in Mathematics Education.

    d. To offer consultancy services to schools.

6. Strategic Priorities of NCTM

- Access and Equity: Advance knowledge about, and infuse in every aspect of mathematics education, a culture of

equity where everyone has access to and is empowered by the opportunities mathematics affords.

- Advocacy: Engage in public and political advocacy to focus policymakers and education decision makers on improving learning and teaching mathematics.
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- It helps learners to become confident and develop inquiring minds.

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